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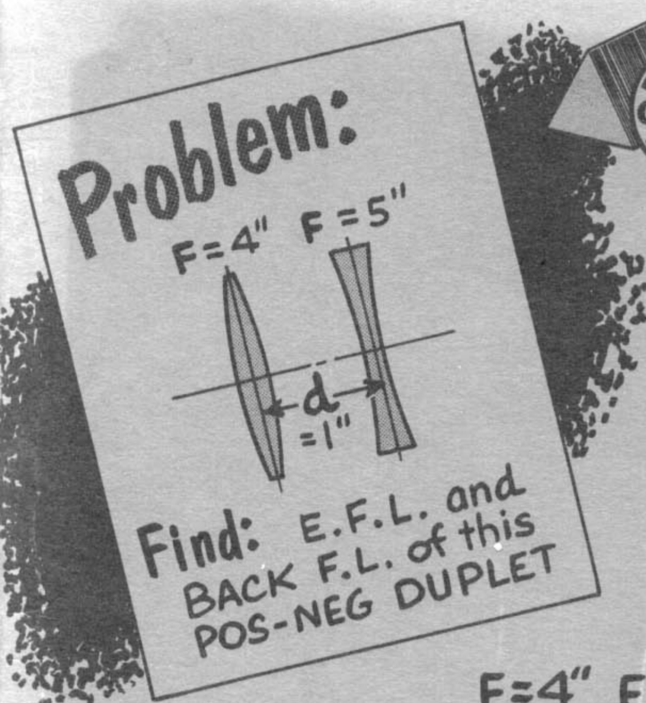
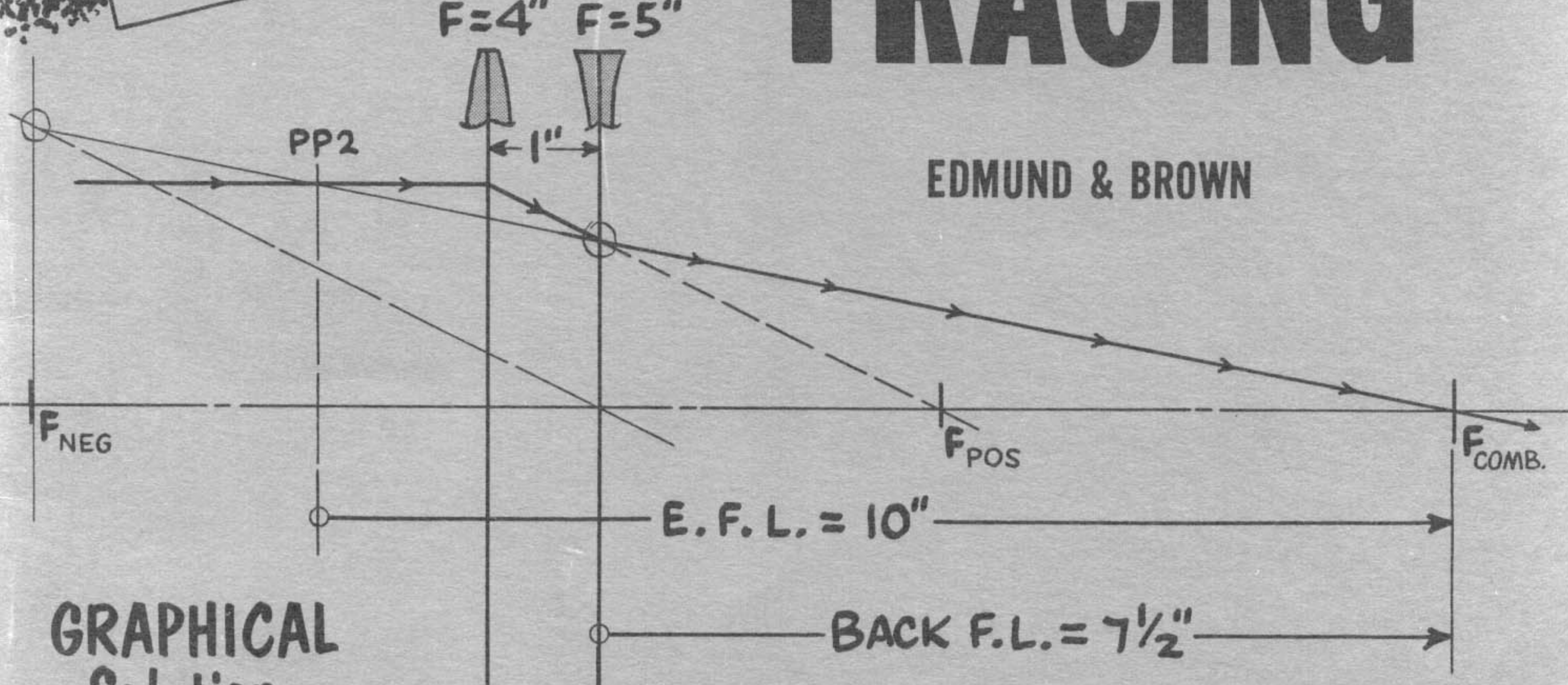
graphical RAY TRACING

EDMUND & BROWN





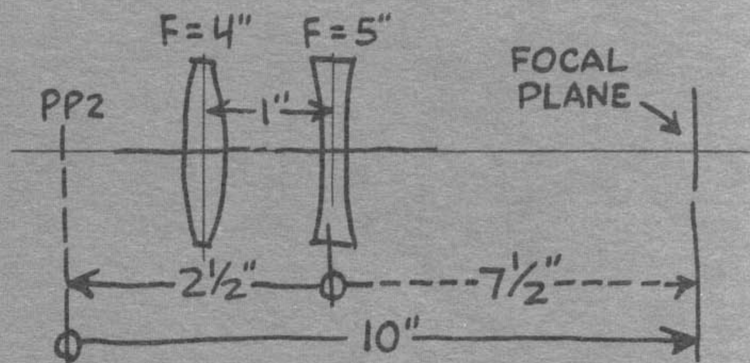
GRAPHICAL Solution

$$\text{E.F.L.} = \frac{\text{POS} \times \text{NEG}}{\text{NEG} - \text{POS} + d}$$

$$\text{E.F.L.} = \frac{4 \times 5}{5 - 4 + 1} = \frac{20}{2} = 10''$$

1+1

$$\text{POSITION of PP2 LEFT from NEG. LENS} = \frac{\text{E.F.L.} \times d}{F_{POS}} = \frac{10 \times 1}{4} = \frac{10}{4} = 2\tfrac{1}{2}''$$



MATH Solution

graphical RAY TRACING

by N.W. Edmund
and Sam Brown

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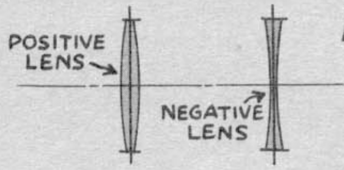
EDMUND SCIENTIFIC CO.
BARRINGTON, NEW JERSEY

IF YOU like optics you will find graphical ray tracing both interesting and instructive. The simple graphical trace combined with a bit of elementary math will give you the approximate first solution to any kind of optical problem involving an object and an image. You can put light rays through multiple lenses almost as easily as through a single element.

Tools needed are the common ones used in any kind of mechanical drawing or drafting, that is, drawing board and T-square. An adjustable triangle is helpful, almost a must for the popular and useful oblique trace. Three or more colors of pencils or ballpoint desk pens will make your work easy on the eyes and easy to understand.

Drawings should be made full-size whenever possible, and if beyond your board, the apertures should be retained full-size, scaling down the longitudinal dimensions only to suit.

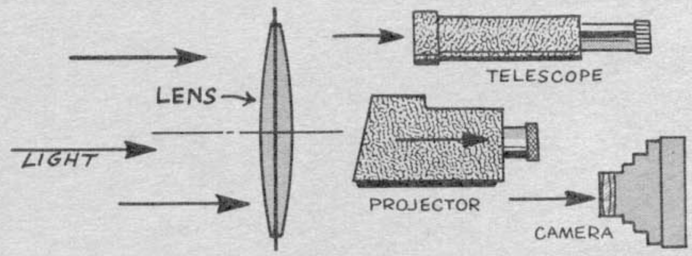
First Pointers



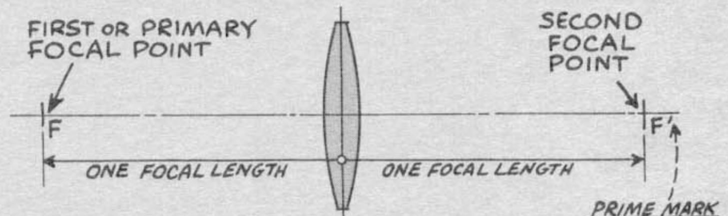
LENSES ARE THIN

A LENS IS ASSUMED TO HAVE NO THICKNESS—IT IS REPRESENTED BY A SINGLE LINE

SINCE THIN LENSES ARE SELDOM OVER 1/4" THICK, NO GREAT ERRORS RESULT FROM NEGLECTING THE KIND OF GLASS AND ITS THICKNESS

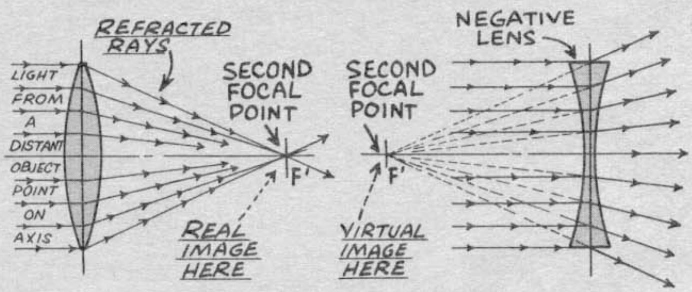


LIGHT COMES FROM THE LEFT *(WHENEVER PRACTICAL)*



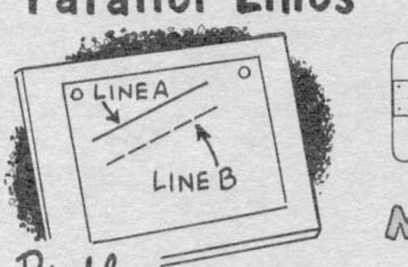
THE FOCAL LENGTH OF A LENS IS THE SAME IN EITHER DIRECTION

PRIME MARK IS OFTEN USED TO MARK SECOND FOCAL POINT

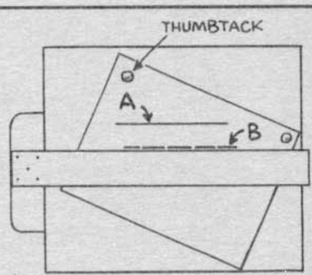


ALL LENSES WORK PERFECTLY. *EXAMPLE IS BASIC CASE WHERE ALL RAYS FROM A DISTANT POINT FOCUS AT ONE F.L.*

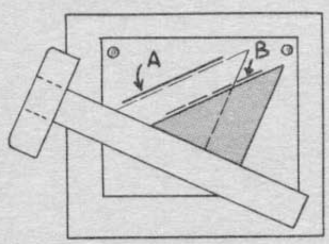
Parallel Lines



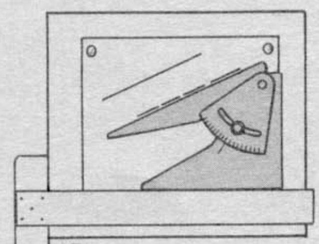
PROBLEM: GIVEN THE SLANT LINE, A, HOW TO DRAW LINE B PARALLEL TO IT



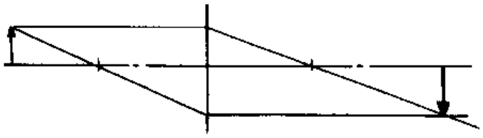
METHOD 1: RE-TACK DRAWING WITH LINE A PARALLEL TO T-SQUARE. YOU CAN THEN DRAW LINE B



METHOD 2: SET A TRIANGLE TO LINE A AND THEN SLIDE IT ALONG HAND-HELD T-SQUARE TO DRAW LINE B



BEST WAY! AN ADJUSTABLE TRIANGLE *(SUCH AS EDMUND NO. 70-789)* HANDLES THE JOB PERFECTLY



Parallel Ray method

THIS most common method of graphical ray tracing requires an off-axis object point which is usually taken as the edge of the object. Applying the three simple rules shown, you can trace light rays from this point, through the lens, to the corresponding point on the image. Only two rays are actually needed to locate the image, but the third is useful as a kind of check or proof. All this is simple geometry, so if at times you don't get a perfect intersection of lines, don't blame it on the "system."

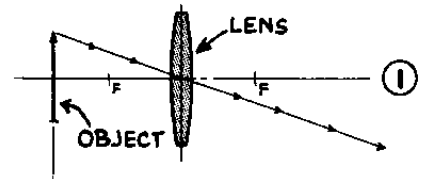
A SECOND LENS. After you locate the image point, any number of additional light rays can be drawn, Fig. 5. Using this idea, you can locate in the cone of light from the first lens, the key rays for a second lens, as shown in Figs. 6, 7 and 8. Of the three initial rays, only the lowermost leaving the first lens parallel is continued through the second lens, Fig. 6. The other two key rays for the second lens must be found by back-tracking from the image point in the manner shown.

OBJECT INSIDE FOCUS. Figs. 10 and 11 show how to handle this situation. You use the same three rules as before, but now the light rays must be extended backwards to locate the virtual image. Anytime you get two or more light rays diverging or spreading after they leave a lens or mirror, you know the image is virtual and can be found by extending the light rays in the opposite direction.

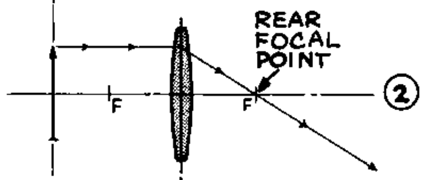
POSITIVE MIRROR. A positive (concave) mirror forms an image in much the same way as a positive lens, except the undeviated ray passes through the center of curvature, which is rule 1 in Fig. 12. The other two rules are the same as for a lens, except the light is reflected instead of being refracted.

The case of an object inside focus, Fig. 13,

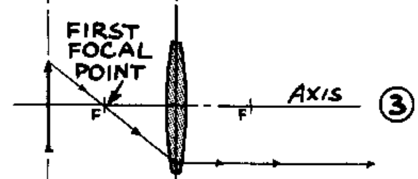
Rule 1: A LIGHT RAY PASSING THROUGH THE CENTER OF LENS IS NOT DEVIATED



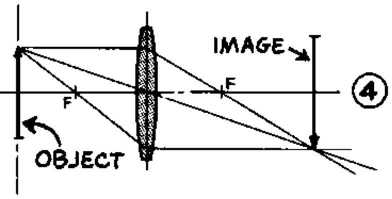
Rule 2: A LIGHT RAY PARALLEL WITH AXIS WILL, AFTER REFRACTION, PASS THRU THE REAR FOCAL POINT



Rule 3: A LIGHT RAY THROUGH THE FIRST FOCAL POINT WILL BE REFRACTED PARALLEL WITH THE AXIS

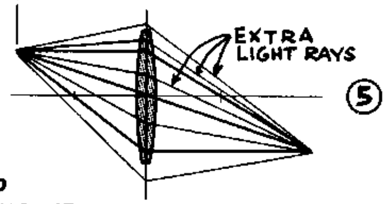


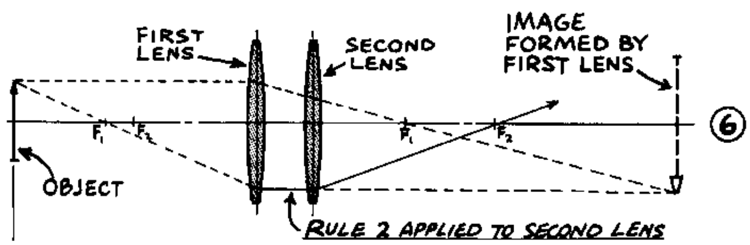
THE INTERSECTION OF ANY TWO OF THE THREE LIGHT RAYS SHOWN WILL LOCATE THE POSITION OF THE IMAGE

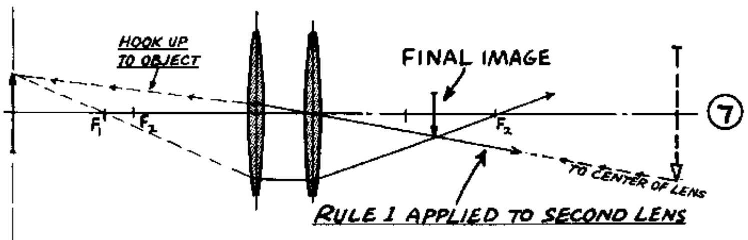


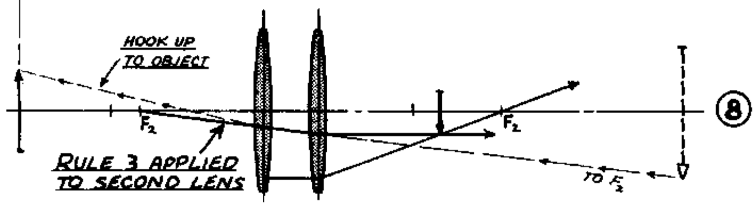
Extra Light Rays

AFTER LOCATING THE IMAGE, YOU CAN "HOOK-UP" ANY NUMBER OF EXTRA RAYS. KEY RAYS FOR A SECOND LENS ARE LOCATED IN THIS MANNER

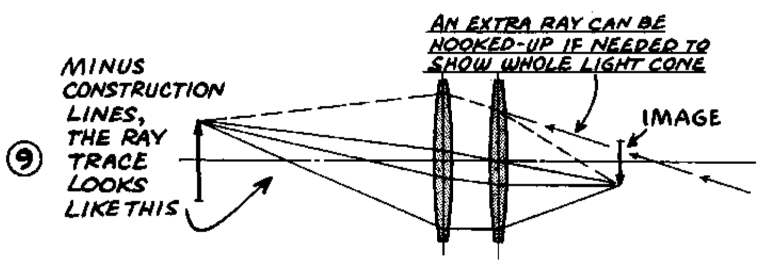








PARALLEL RAY METHOD FOR TWO LENSES

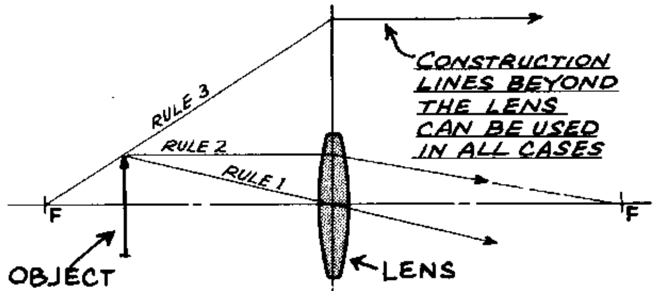


WHOLE CONE OF LIGHT RAYS

PARALLEL RAY METHOD

(10) OBJECT AT LESS THAN ONE FOCAL LENGTH FROM A POSITIVE LENS

THE LIGHT RAYS ARE DRAWN IN THE USUAL MANNER, BUT ARE SEEN TO BE SPREADING AND CANNOT INTERSECT TO FORM AN IMAGE



(11) *BACKWARD EXTENSION OF THE RAYS WILL INTERSECT TO LOCATE THE IMAGE WHICH IS VIRTUAL*

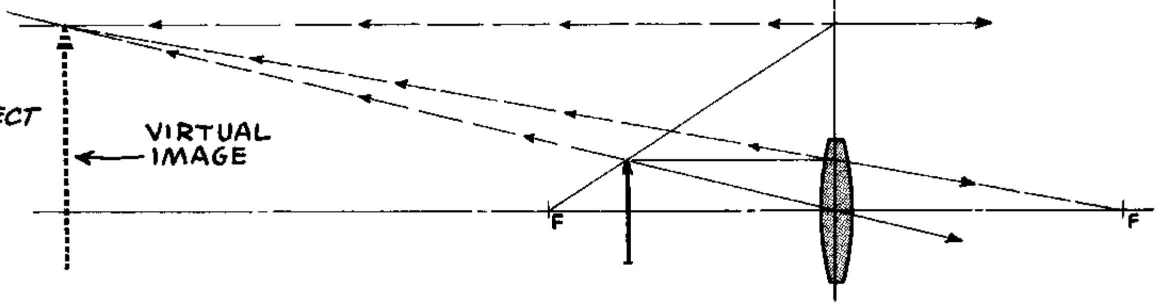
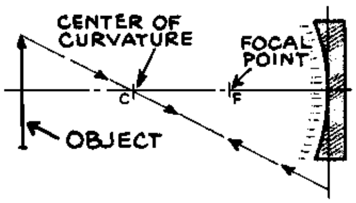


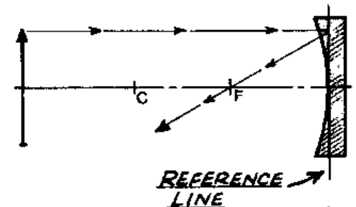
IMAGE FORMATION BY A (CONCAVE) POSITIVE MIRROR

(12)

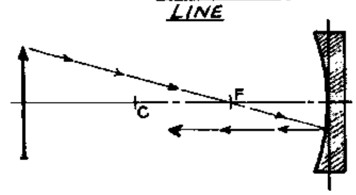
Rule 1: A RAY THRU THE CENTER OF CURVATURE IS NOT DEVIATED - *IT IS REFLECTED BACK OVER THE SAME PATH*



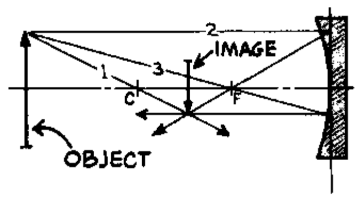
Rule 2: A RAY PARALLEL TO THE AXIS WILL BE REFLECTED THRU THE FOCAL POINT



Rule 3: A RAY PASSING THRU THE FOCAL POINT WILL BE REFLECTED PARALLEL TO AXIS



THE INTERSECTION OF ANY TWO OF THE THREE RAYS SHOWN WILL LOCATE THE POSITION OF IMAGE



is also similar to the same construction for a lens. Notice that light rays in a mirror trace are bent at a straight reference line drawn through the vertex of the mirror; this is done to preserve the geometrical relationship which is the basis for this kind of graphical ray tracing. Actually you are not dealing with lenses or mirrors at all but only with similar triangles.

NEGATIVE LENSES AND MIRRORS. These cases are shown in Fig. 14 and 15 and are complete and self-explanatory. The negative lens or mirror always produces a virtual image, always reduced in size, always on the opposite side of the lens or mirror to the normal position of a real image formed by a positive lens or mirror.

VIRTUAL PRIMARY IMAGE. There are two common cases where the first of two lenses will form a virtual image to the left of a second lens. These cases are shown in Figs. 16 and 18. Whenever this situation occurs, you can use the primary image formed by the first lens as a real object for the second lens in the sense you can treat the second lens independently.

(13) OBJECT AT LESS THAN ONE F.L. FROM A CONCAVE MIRROR

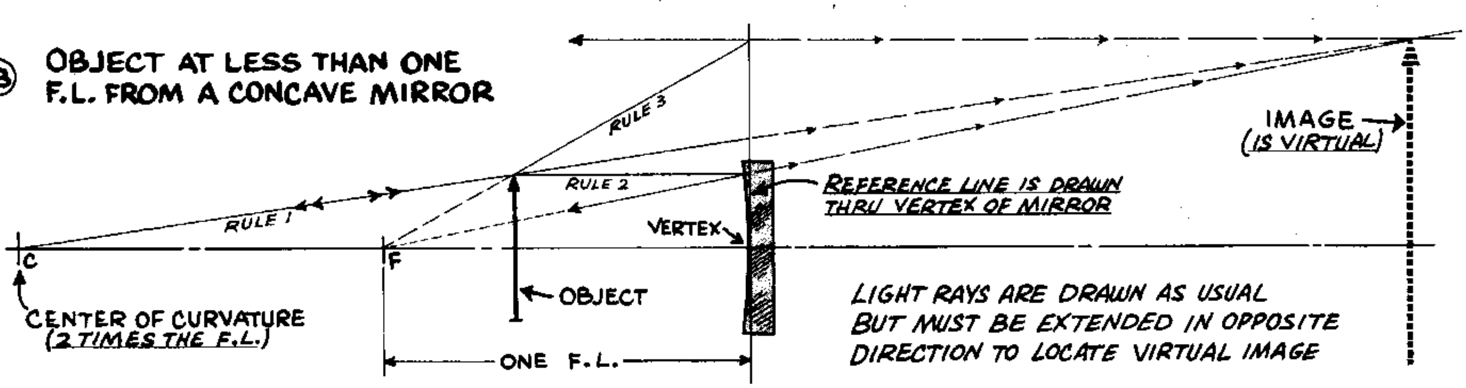
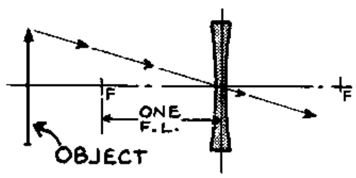


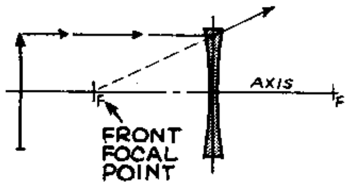
IMAGE FORMATION BY A NEGATIVE LENS with OBJECT AT ANY DISTANCE

(14)

Rule 1: A LIGHT RAY PASSING THRU THE CENTER OF LENS IS NOT DEVIATED

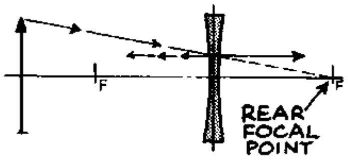


Rule 2: A LIGHT RAY PARALLEL TO THE AXIS WILL, AFTER REFRACTION, APPEAR TO COME FROM THE FRONT FOCAL POINT

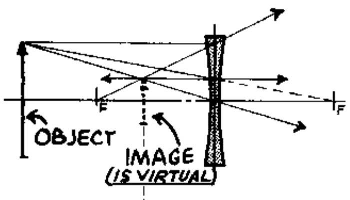


Rule 3: A RAY AIMED AT THE REAR FOCAL POINT WILL BE REFRACTED PARALLEL TO THE AXIS

A BACKWARD EXTENSION OF THIS RAY WILL PASS THRU THE IMAGE POINT



THE INTERSECTION OF ANY TWO OF THE THREE RAYS SHOWN WILL LOCATE THE POSITION OF THE IMAGE

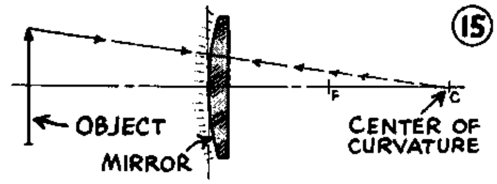


PARALLEL RAY METHOD

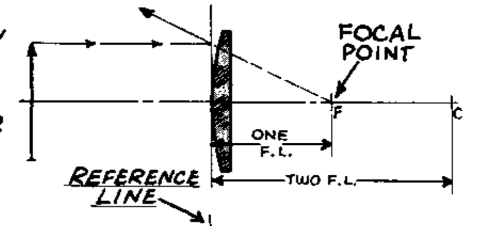
IMAGE FORMED BY A NEGATIVE (CONVEX) MIRROR

(15)

Rule 1: A RAY AIMED AT THE CENTER OF CURVATURE IS NOT DEVIATED

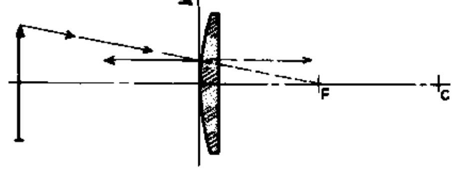


Rule 2: A RAY PARALLEL TO AXIS WILL, AFTER REFLECTION, *APPEAR* TO COME FROM THE FOCAL POINT

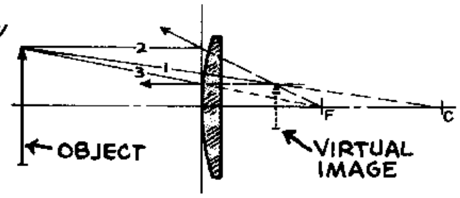


Rule 3: A RAY AIMED AT THE FOCAL POINT WILL BE REFLECTED PARALLEL TO AXIS

A BACKWARD EXTENSION OF THIS RAY WILL PASS THRU THE IMAGE POINT

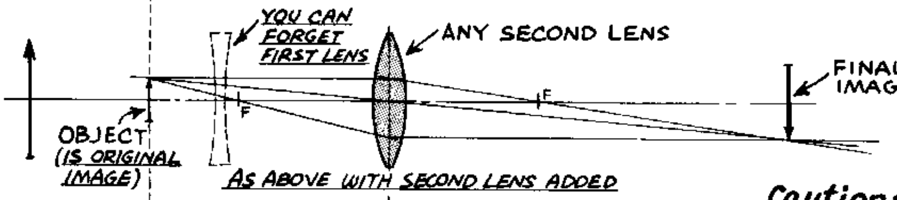


THE INTERSECTION OF ANY TWO RAYS WILL LOCATE THE IMAGE POSITION

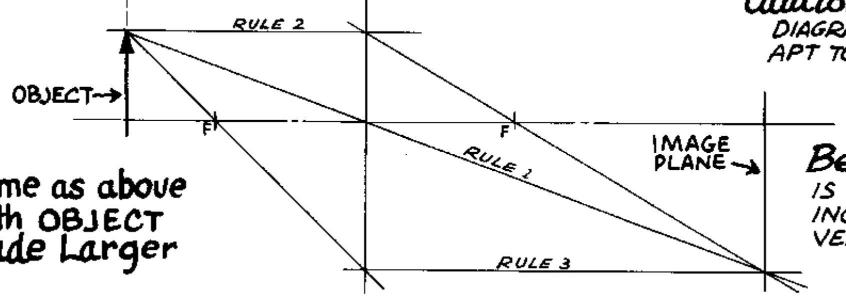


(16) TWO LENSES WITH VIRTUAL IMAGE TO LEFT

Rule: A VIRTUAL IMAGE TO THE LEFT OF A SECOND LENS CAN BE USED INDEPENDENTLY AS A REAL OBJECT FOR THE SECOND LENS



Caution: *LONG, SKINNY DIAGRAMS LIKE THIS ARE APT TO CAUSE ERRORS*



(17) Same as above with OBJECT Made Larger

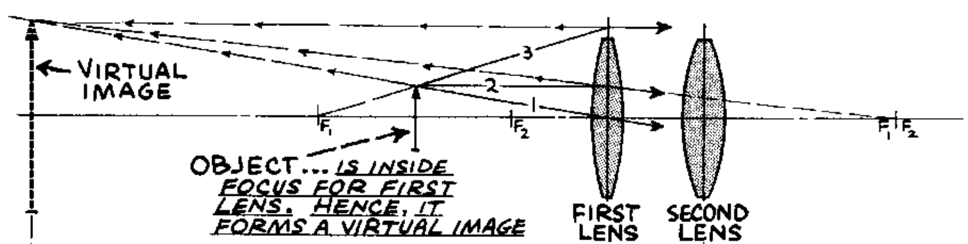
Better Accuracy *IS OBTAINED BY INCREASING THE VERTICAL SCALE*

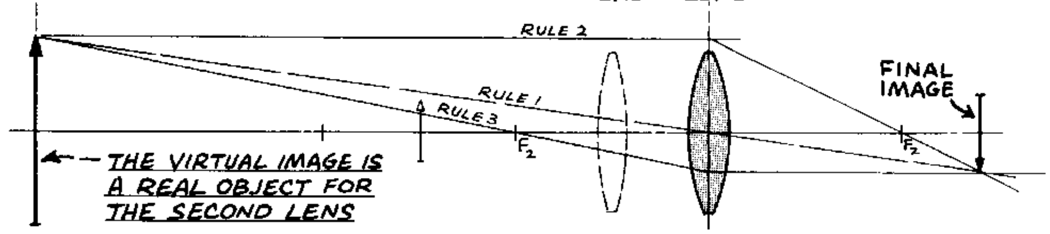
Second Example

TWO LENSES WITH VIRTUAL IMAGE TO LEFT

(18)

THIS SHOWS THE ONE OTHER SITUATION WHERE A VIRTUAL OBJECT IS TO THE LEFT OF A SECOND LENS





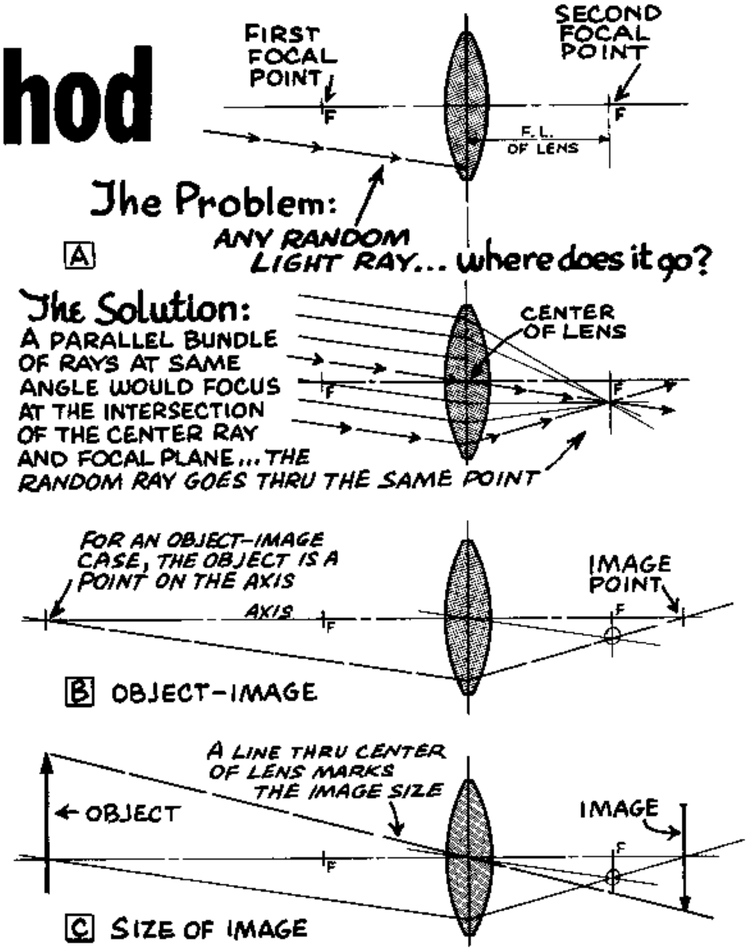
Oblique Ray method

THE OBLIQUE ray method has the useful feature that you can start out with any random ray and trace this ray through any number of lenses. If you are working with an object target, it must be a point on the axis. This, of course, is just the opposite of the parallel ray method where the object point must be off the axis. A light ray from any axial object point will form an image at any point where it again crosses or appears to cross the optical axis. The "appears to cross" applies to negative lenses and other cases where the image is virtual.

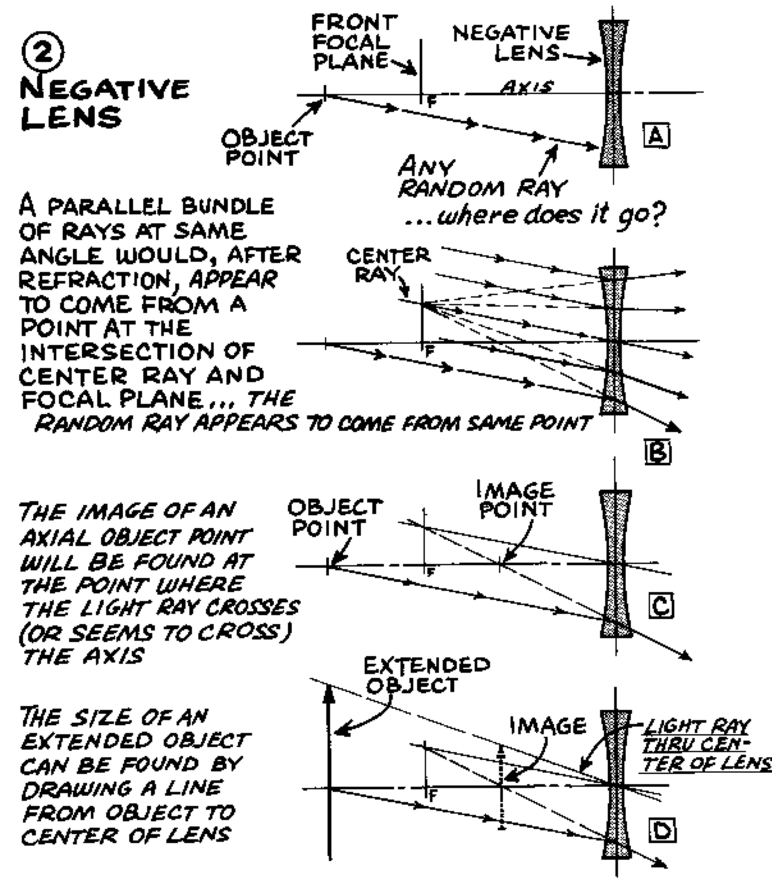
SINGLE POSITIVE LENS. You start out with any random ray. You don't know where this ray is going, but, if you had a bundle of rays parallel to the random ray, they would all come to a focus at the second focal plane, as shown at A in Fig. 1. Included in this bundle is the undeviated ray passing through the center of the lens. Included also is the random ray itself. Hence, the oblique ray construction is just a matter of drawing a line through the center of the lens, parallel to the random ray. At the point where this ray intersects the second focal plane, you establish a point through which the random ray must pass. Extending the light ray, you will find it cuts across the axis, locating the image point of the corresponding axial object. It is worth noting again that the intersection of the undeviated ray with the focal plane marks a point through which the random ray must pass--it is not the location of an image. The single-line trace does not show the size of the image. This is readily determined with a line drawn from object through the center of lens, as at C, Fig. 1.

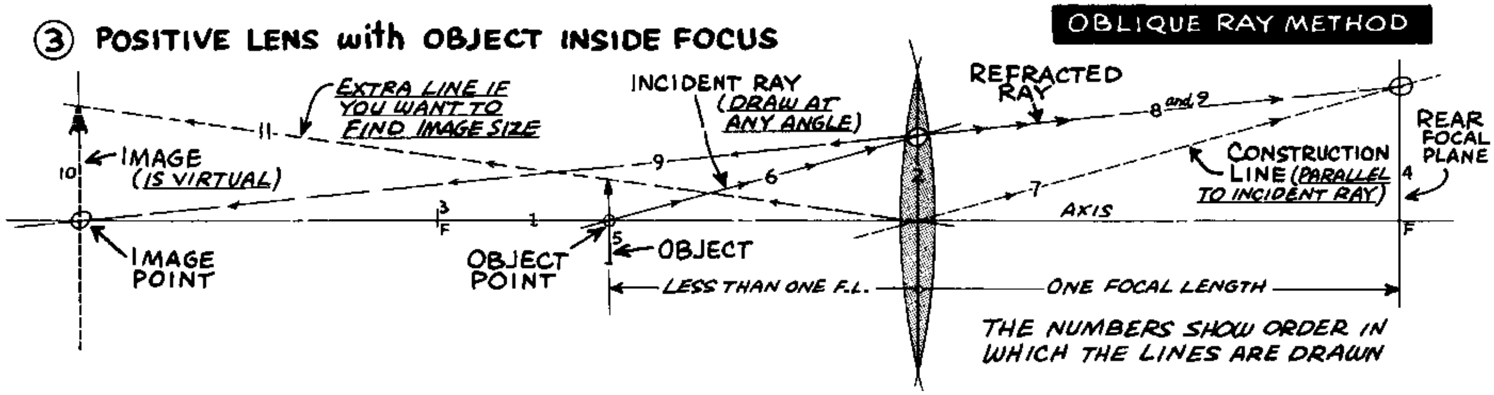
NEGATIVE LENS. A negative lens is a diverging lens--it spreads the light rays instead of bringing them to a focus. Thus, you have the general case where refracted light rays appear to come from a virtual image. An incident bundle of parallel rays will appear to come from the front focal plane, as shown at B, Fig. 2. Here again you are interested only in the thru-the-center-of-lens ray, which is the one that locates the guide point through which the refracted ray must pass, Fig. 20. As before, an extra line is needed to find the image size of an extended object, Fig. 2D.

OBJECT INSIDE FOCUS. An object at less than



(1) OBLIQUE RAY METHOD for a POSITIVE LENS

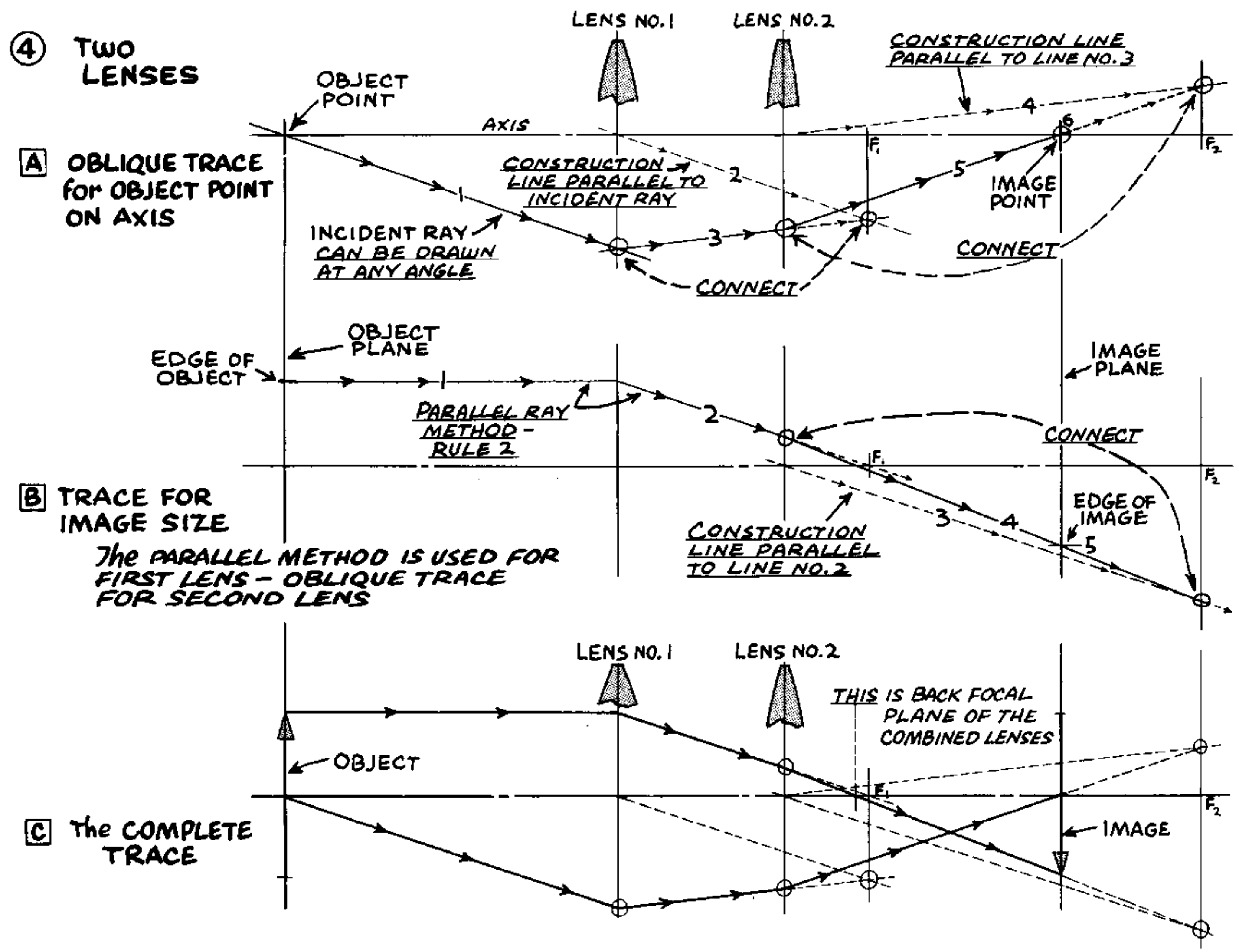




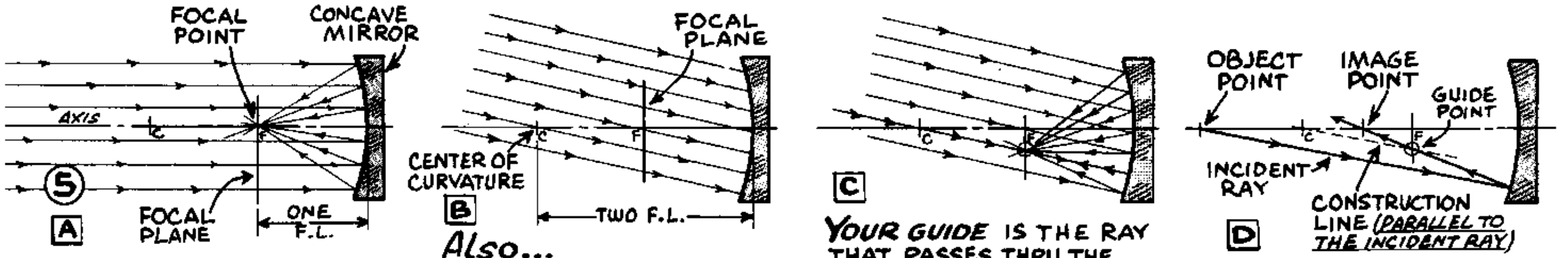
one focal length from a lens produces a virtual image on the same side of the lens as the object. A typical case is shown in Fig. 3. Ray tracing by the oblique method procedes just the same as before, the only difference being that the refracted ray will be seen to be diverging and must be extended back toward the object in order to locate the axial image point.

TRACING TWO LENSES. Two or more lenses are easily traced by the oblique ray method. You start out with a random ray incident on the first lens at any angle or height, as shown in Fig. 4A. You don't know where this ray is going, but if you draw a parallel construction line, you know the ray will be refracted to the point where the construction line intersects the focal plane. Line 3 in diagram 4A is the refracted ray for the first lens, drawn as described.

The procedure is repeated for the second lens. The end result locates the image point of an axial object. If the object is extended in size, an additional ray must be traced to get the image size. This extra ray is started on its way by the



OBLIQUE RAY METHOD ....how it works with MIRRORS

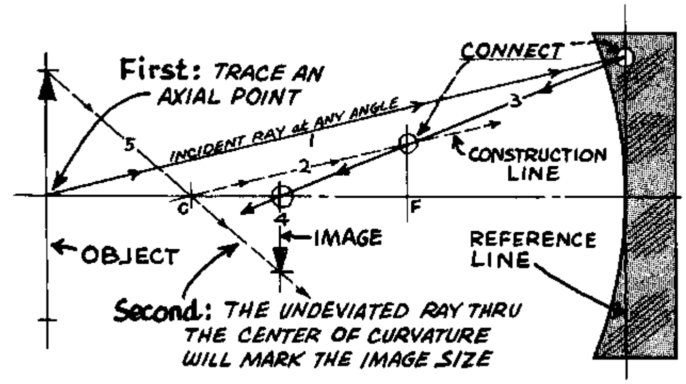


A BUNDLE OF PARALLEL LIGHT RAYS FROM A DISTANT AXIAL POINT WILL COME TO A FOCUS ON THE AXIS AT THE FOCAL POINT OF THE MIRROR
A POSITIVE (CONCAVE) MIRROR IS ASSUMED

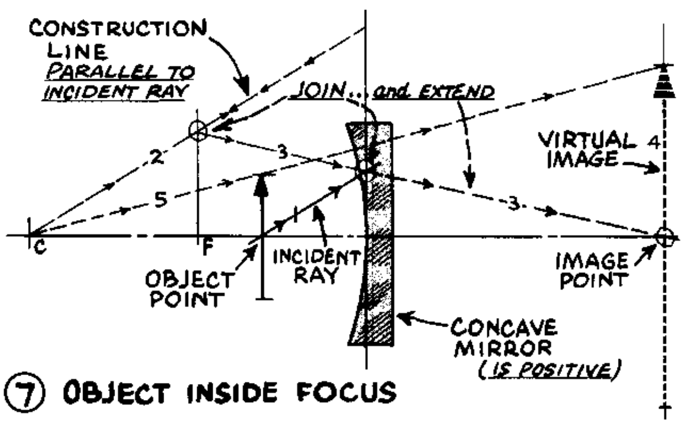
Also...
A PARALLEL BUNDLE OF LIGHT RAYS FROM A DISTANT OFF-AXIS POINT WILL COME TO A FOCUS AT SOME POINT ON THE FOCAL PLANE *...but where?*

YOUR GUIDE IS THE RAY THAT PASSES THRU THE CENTER OF CURVATURE. THIS RAY IS NOT DEVIATED *...IT GOES TO THE MIRROR AND COMES RIGHT BACK AGAIN.* THIS RAY COMES TO A FOCUS AT THE POINT WHERE IT CROSSES THE FOCAL PLANE

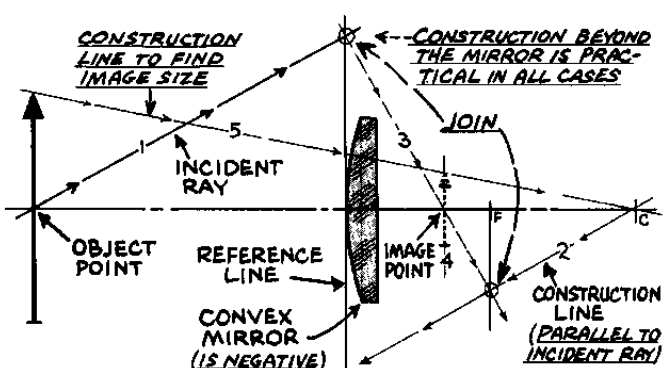
YOU DON'T KNOW WHERE THE INCIDENT RAY IS GOING BUT IF YOU DRAW A LINE PARALLEL TO IT FROM **C**, YOU WILL LOCATE A POINT ON THE FOCAL PLANE THRU WHICH THE REFLECTED RAY MUST PASS



⑥ **IMAGE SIZE of an EXTENDED OBJECT**



⑦ **OBJECT INSIDE FOCUS**



⑧ **NEGATIVE (CONVEX) MIRROR with OBJECT AT ANY DISTANCE**

parallel ray method, Fig. 4B, and is then continued with an oblique trace for lens No. 2. The complete trace is shown at C. The parallel ray from the edge of the object also represents the path of a ray from a distant axial point; hence, where this ray crosses the axis locates the back or second focal plane of the combination.

OBLIQUE TRACE FOR MIRRORS. The oblique trace for a mirror is much the same as for a lens with the important difference that the undeviated ray passes through the center of curvature. The theory of the method is shown in Fig. 5 at A, B and C; the trace itself is shown at D. As with a lens, you draw a construction line parallel to the incident ray. At the point where this line crosses the focal plane, you establish a guide point through which the reflected ray must pass. Continuing this ray to the axis locates the axial image point. The case shown in Fig. 5 is for a positive (concave) mirror with object at more than one focal length. Fig. 6 shows the extra line needed to locate the image size of an extended object.

OBJECT INSIDE FOCUS. An object at less than one focal length from a positive mirror will produce a virtual image, which appears to come from behind the glass, Fig. 7. The oblique construction is much the same as before, except the actual reflected ray is seen to be divergent and must be extended in the opposite direction to locate the virtual image behind the mirror.

NEGATIVE MIRROR. A negative (convex) mirror always produces a virtual image, reduced in size and appearing to come from behind the mirror. The oblique ray trace is shown in Fig. 8. The actual reflected ray is not shown, but only its extension--line No. 3--which locates the axial image point where is crosses the axis.

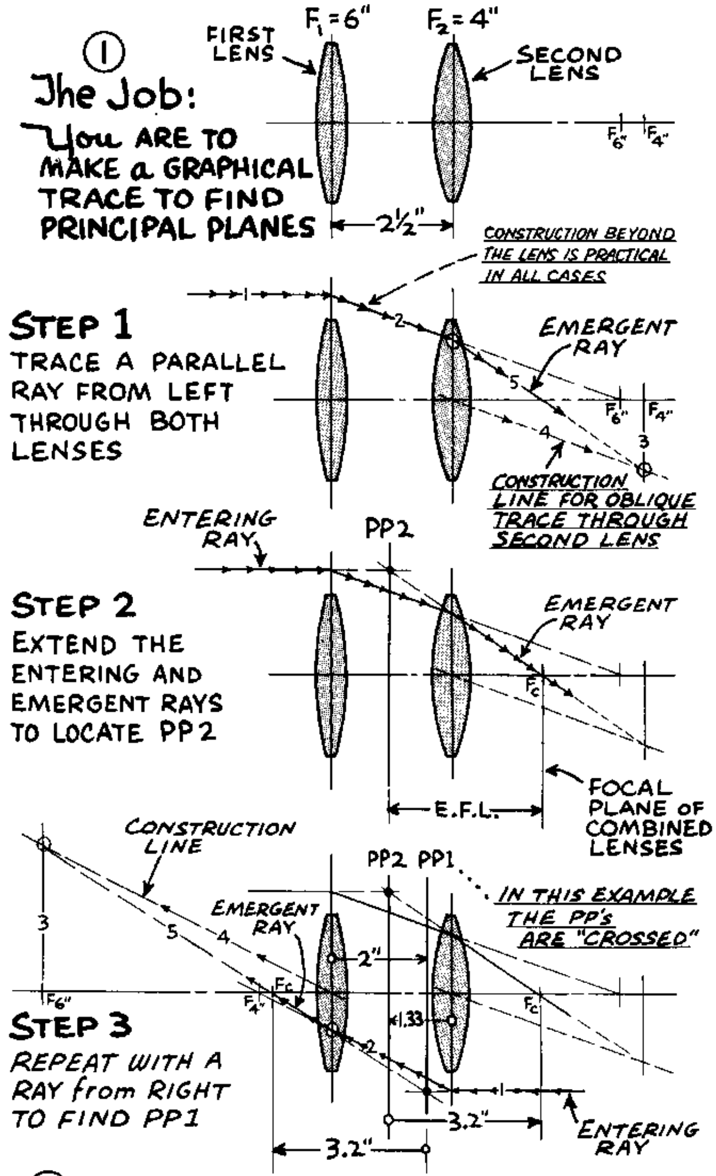
Principal Planes

PRINCIPAL PLANES are imaginary planes in a lens or lens system from which measurements are made. Ordinarily, measuring distances from the center of a thin lens is accurate enough, but as lenses become thicker, a measurement from the proper PP is more accurate. PP1 is associated with any dimension to the front of the lens; PP2 is the reference line for any measurement on the image side. The PP's of simple lenses can be located close enough by eye, as shown in boxed diagram below.

PP's OF TWO LENSES. The principal planes of two lenses can be found by a graphical ray trace. A typical example is shown in Fig. 1. First, you run in a parallel-with-axis ray from the left and take it through the two lenses by the tracing methods already described, as shown in Step 1. If the final emergent ray is projected backwards, it will intersect an extension of the original ray, as shown in Step 2, the intersection being the location of PP2. PP1 is found by running in a similar ray trace from the right, Step 3.

In connection with the graphical trace, it is often useful to calculate the e.f.l. of the two lenses, using the needed formula from Fig. 6 table. The PP positions also can be calculated, Fig. 7, and the combined math-graph operation provides an assuring double-check.

USING THE PP's. Knowing the PP's of a two-lens system, you can treat the combined lenses as a single lens. Any distance to the image is measured from PP2; any distance to the object is measured from PP1. Fig. 3 is an example. Here the PP's are crossed, but this does not alter the procedure in tracing light rays--you



2 MATH CHECK:

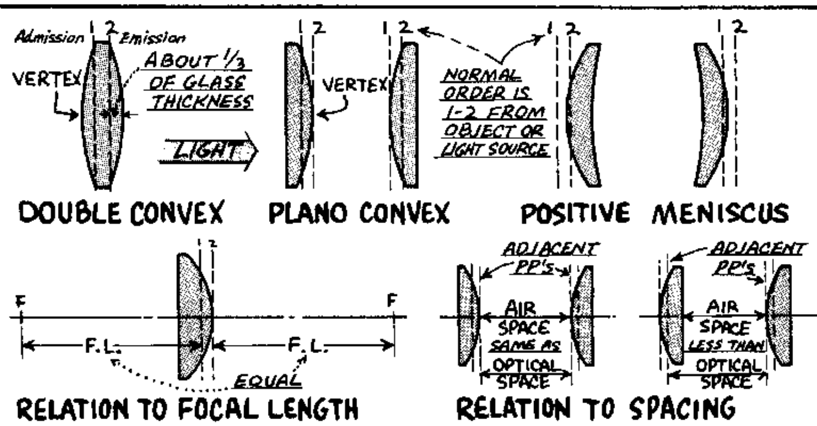
$$\text{E.F.L.} = \frac{6 \times 4}{6+4-2.5} = \frac{24}{7.5} = 3.2''$$

PP1... *RIGHT, from CENTER OF 1st LENS* $= \frac{3.2 \times 2.5}{4} = \frac{8}{4} = 2''$

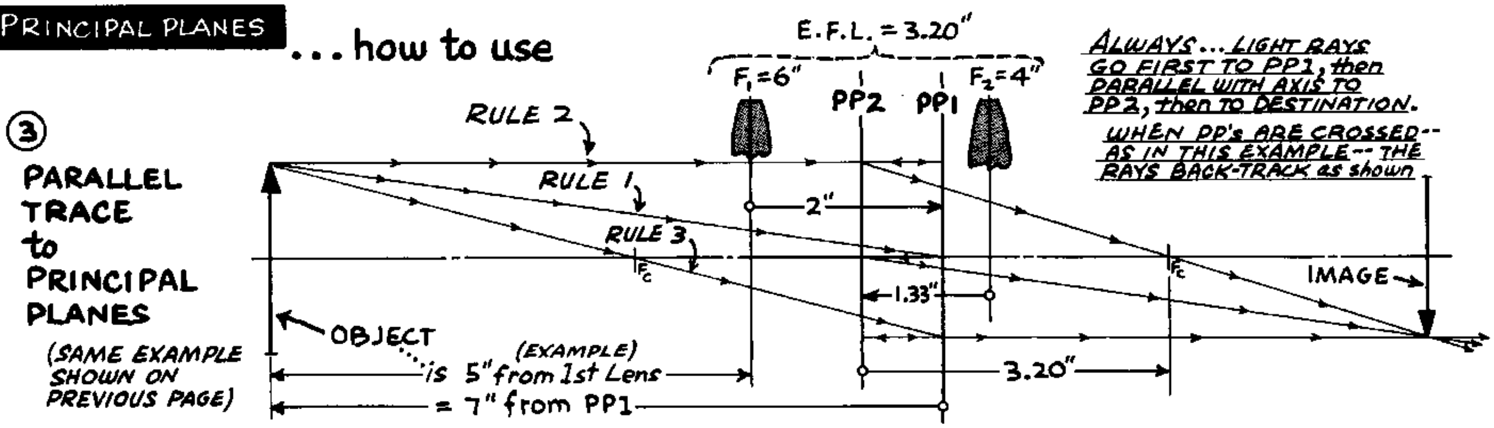
PP2... *LEFT, from CENTER OF 2nd LENS* $= \frac{3.2 \times 2.5}{6} = \frac{8}{6} = 1.33''$

PRINCIPAL PLANES
of SIMPLE POSITIVE LENSES

Usually it is close enough to set off the focal length or other measurement from the center of a lens, but if you want to be exact, the proper measuring points are the principal planes, as shown. The PP's are numbered 1-2 from the object side; the numbers interchange when the lens is faced in opposite direction, as can be seen in center top diagram.

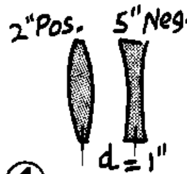


PRINCIPAL PLANES ... how to use



④ Calculate EFL and PP's

POS-NEG with NEG Longest



$$EFL\ (f_c) = \frac{POS \times NEG}{NEG - POS + d} = \frac{2 \times 5}{5 - 2 + 1} = \frac{10}{4} = 2\tfrac{1}{2}''\ \text{POS.}$$

$$PP1 = \frac{F_c \times d}{F_2} = \frac{2.5 \times 1}{5} = \frac{2.5}{5} = .5''\ \text{(LEFT)}$$

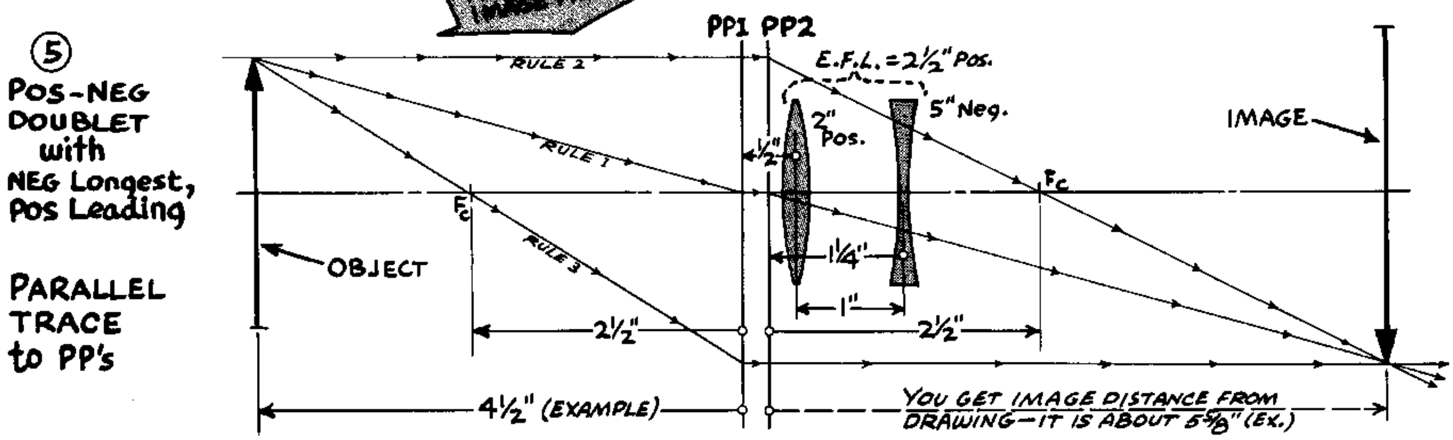
$$PP2 = \frac{F_c \times d}{F_1} = \frac{2.5 \times 1}{2} = \frac{2.5}{2} = 1.25''\ \text{(LEFT)}$$

(PP's ARE NORMAL 1, 2)

USE THIS DATA FOR ANY OBJECT-IMAGE PROBLEM

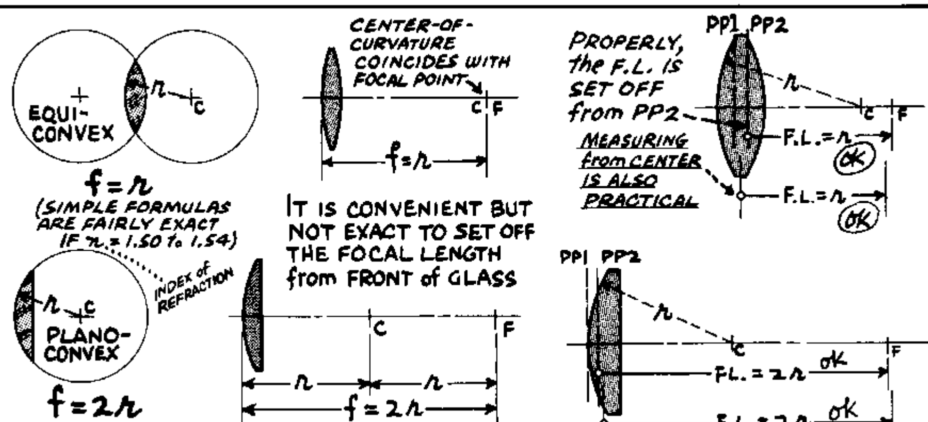
go first to PP1, then parallel with axis to PP2, and finally to the destination point. Usually an object-image trace is made by the parallel ray method, which is shown. Fig. 5 is another example, and this time you will note the PP's are in normal 1, 2 order.

MATH FORMULAS. Calculating the equivalent focal length of a two-lens system is one of the most common operations in optics. The tables on opposite page eliminate all sign conventions and algebra by giving a separate formula for each of the combinations possible with two lenses.



Lens Diagrams

The f. l. of a simple lens is increased by glass thickness and decreased by a refractive number higher than 1.50. The way this works out, you can apply the simple formulas shown to get f. l. with a good degree of accuracy, assuming crown glass (index about 1.52) for most lenses.



⑥ EQUIVALENT FOCAL LENGTH of TWO LENSES

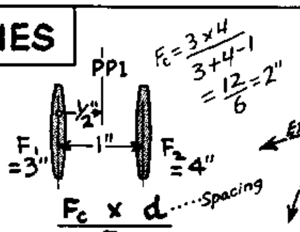
	TWO POSITIVE	TWO NEGATIVE	POSITIVE and NEGATIVE: NEGATIVE is *LONGER*	POSITIVE and NEGATIVE: NEG. is *SHORTER* F.L.
BASIC FORMULA	$E.F.L. = \frac{F \times F}{F+F-d}$ (F.L. of first lens × F.L. of second lens; d = spacing)	$E.F.L._{(NEGATIVE)} = \frac{F \times F}{F+F+d}$	$E.F.L. = \frac{POS \times NEG}{NEG-POS+d}$	$E.F.L._{(NEGATIVE)} = \frac{POS \times NEG}{POS-NEG-d}$
Examples: CONTACT	F=6" F=4"; d=0 (NEARLY); F_c (F.L. of combined lenses) $= \frac{6\times4}{6+4-0} = \frac{24}{10} = 2.4"$	F=6" F=4"; d=0 (NEARLY); $F_c = \frac{6\times4}{6+4+0} = \frac{24}{10} = 2.4"$ NEG.	F=2" POS, F=4" NEG; d=0 (NEARLY); $F_c = \frac{2\times4}{4-2+0} = \frac{8}{2} = 4"$	F=4" POS, F=2" NEG; d=0 (NEARLY); $F_c = \frac{4\times2}{4-2-0} = \frac{8}{2} = 4"$ NEG.
MODERATE SPACING	6" 4"; d=2"; $F_c = \frac{6\times4}{6+4-2} = \frac{24}{8} = 3"$	6" 4"; d=2"; $F_c = \frac{6\times4}{6+4+2} = \frac{24}{12} = 2"$ NEG.	2" POS, 4" NEG; d=1"; $F_c = \frac{2\times4}{4-2+1} = \frac{8}{3} = 2.66"$	4" POS, 2" NEG; d=1"; $F_c = \frac{4\times2}{4-2-1} = \frac{8}{1} = 8"$ NEG.
SPACING LIMIT (IF ANY)	6" 4"; d=10" (6+4); *LIMIT*: $d = F_1 + F_2$; $F_c = \frac{6\times4}{6+4-10} = \frac{24}{0} = \infty$; *THE F.L. OF COMBINED LENSES IS INFINITE. THIS IS SYSTEM OF ASTRO TELESCOPE*	NO SPACING LIMIT. NO CHANGE IN FORMULA. *E.F.L. ALWAYS NEGATIVE and DECREASES WITH INCREASED SPACING*	NO SPACING LIMIT. NO CHANGE IN FORMULA. *THE E.F.L. (F_c) IS ALWAYS POSITIVE and BECOMES LESS WITH INCREASED SPACING*	4" 2"; d=2" (4-2); *LIMIT*: d = POS−NEG; $F_c = \frac{4\times2}{4-2-2} = \frac{8}{0} = \infty$; *THE F.L. IS INFINITE. THIS IS SYSTEM OF A GALILEAN TELESCOPE*
SPACING OVER THE LIMIT	change FORMULA: $E.F.L._{(NEGATIVE)} = \frac{F \times F}{d-(F+F)}$; 6" 4"; d=12"; $F_c = \frac{6\times4}{12-(6+4)} = \frac{24}{2} = 12"$ NEG.; *COMPOUND SYSTEM IS BASIS FOR PROJECTION and THE MICROSCOPE*	*NO LIMIT* **Sample:** 6" 4"; d=10"; $F_c = \frac{6\times4}{6+4+10} = \frac{24}{20} = 1.2"$ NEG.	*NO LIMIT* **Sample:** 2" POS, 4" NEG; d=10"; $F_c = \frac{2\times4}{4-2+10} = \frac{8}{12} = .66"$	change FORMULA: $E.F.L._{(POSITIVE)} = \frac{POS \times NEG}{NEG+d-POS}$; 4" POS, 2" NEG; d=3"; $F_c = \frac{4\times2}{2+3-4} = \frac{8}{1} = 8"$; *COMPOUND SYSTEM USED FOR TELEPHOTO LENSES and BARLOW SYSTEMS*

NOTE: ALL FORMULAS ARE ARRANGED TO BE WORKED BY SIMPLE ARITHMETIC... MINUS MEANS ONLY SUBTRACTION

⑦ PRINCIPAL PLANES

Standard FORMULAS:

DISTANCE from FIRST LENS (or PP1 of First Lens) TO PP1 of COMBINATION $= \frac{F_c \times d}{F_2}$ (d = spacing)

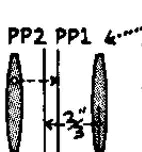


Example: $\frac{2 \times 1}{4} = \frac{2}{4} = \frac{1}{2}"$

DISTANCE from SECOND LENS (or PP2 of Second Lens) TO PP2 of COMBINATION $= \frac{F_c \times d}{F_1} = \frac{2 \times 1}{3} = \frac{2}{3}"$

THESE FORMULAS APPLY TO ALL CASES

IF THE TWO LENSES ARE IDENTICAL, THE PP's WILL BE SYMMETRICAL



THE PP's ARE "CROSSED." THIS IS ALWAYS THE CASE WITH POSITIVE LENSES UNLESS SPACING IS VERY CLOSE

PP DIRECTION ... *spacing (d) less than Limit*

	TWO POS.	TWO NEG.	POSITIVE and NEGATIVE: NEG. Longer, POS. First	NEG. Longer, NEG. First	NEG. Shorter, POS. First	NEG. Shorter, NEG. First
PP1	RIGHT	RIGHT	LEFT	RIGHT	RIGHT	LEFT
PP2	LEFT	LEFT	LEFT	RIGHT	RIGHT	LEFT
Order	USUALLY CROSSED	NORMAL	NORMAL	NORMAL	CROSSED	CROSSED

...spacing MORE than limit

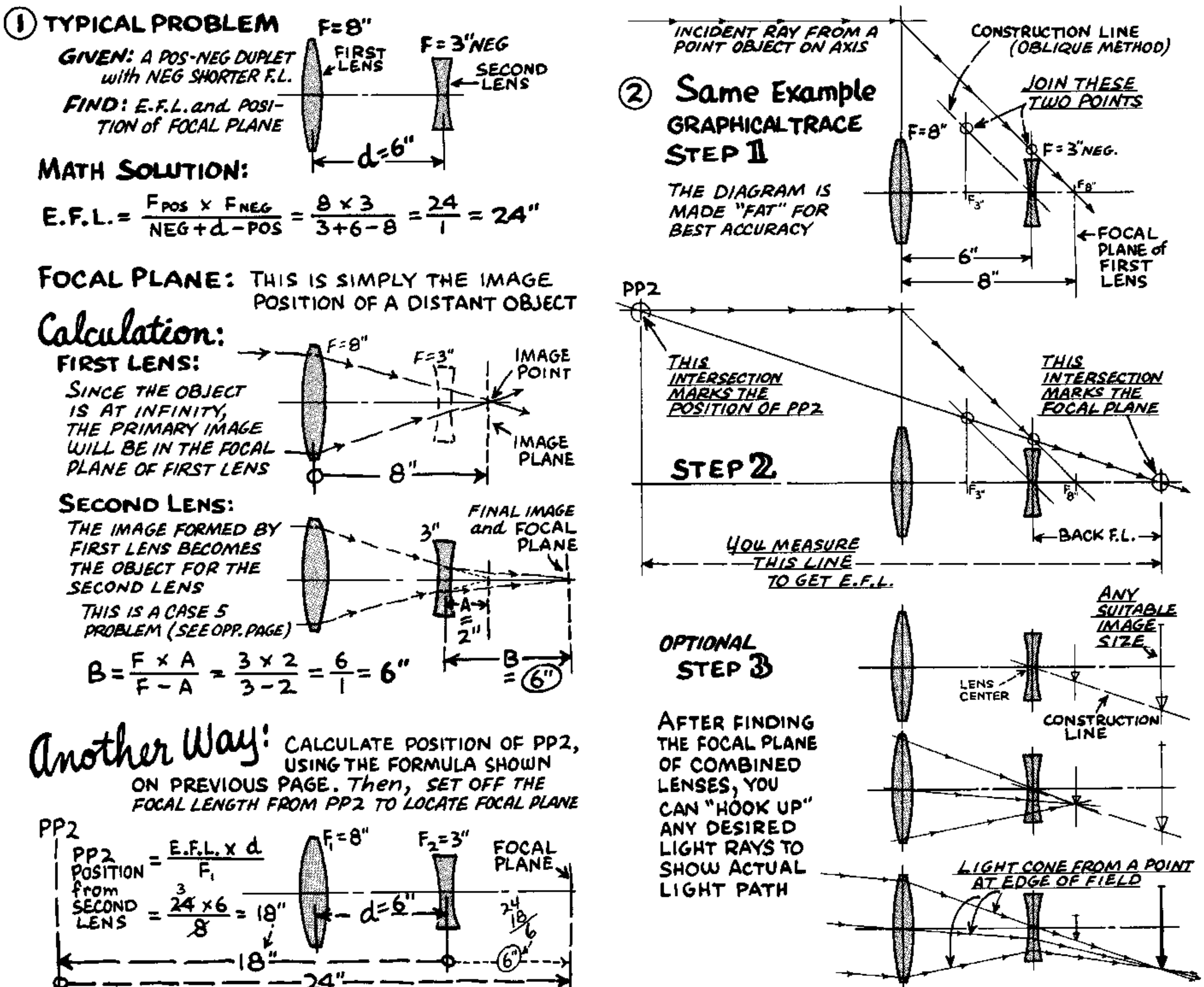
	TWO POS.	TWO NEG.	POSITIVE and NEGATIVE: NEG. Longer, POS. First	NEG. Longer, NEG. First	NEG. Shorter, POS. First	NEG. Shorter, NEG. First
PP1	LEFT	—	—	—	LEFT	RIGHT
PP2	RIGHT	AS ABOVE	AS ABOVE	AS ABOVE	LEFT	RIGHT
Order	NORMAL	—	—	—	NORMAL	NORMAL

OBJECT-IMAGE MATH

GRAPHICAL RAY TRACING inevitably reveals that math is faster and more accurate for most problems. Perhaps the best feature of the graphical trace is the assurance you get from actually putting the light rays through the various lenses. A combination of math and graph is often the best solution. The math itself is simple, being confined to the elementary thin-lens equations. The general idea is that you know the f.l. of the lens and the position of the object from the lens. The problem, then, is to find the position of the image. Fig. 3 table on opposite page covers all of the common situations. Strangely, the formulas become useless for the common case of an object at infinity, but on the other hand such a math solution is never needed--if the object is at infinity, the image will be exactly one focal length from the lens.

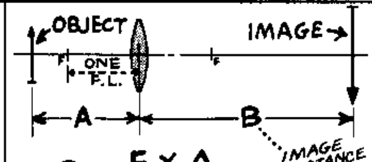
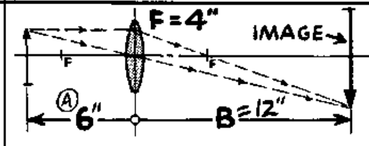
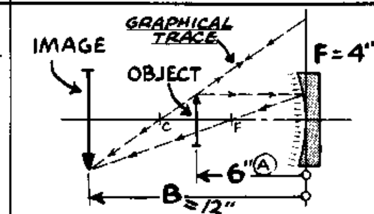
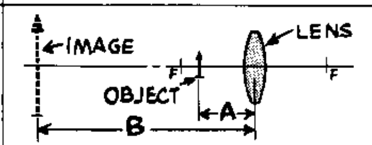
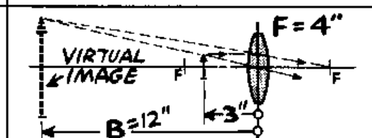
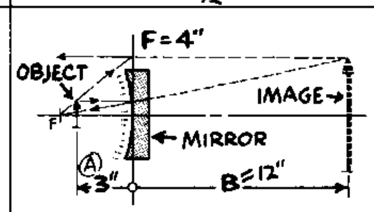
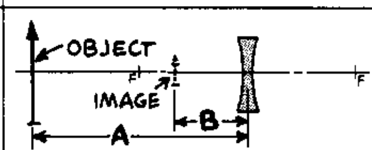
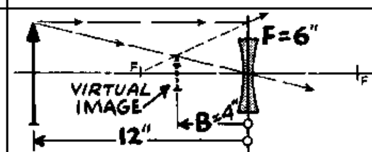
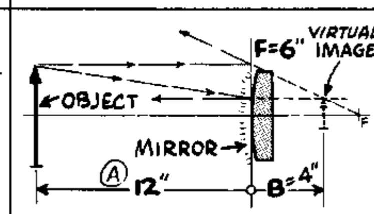
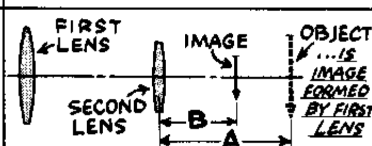
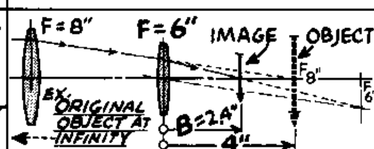
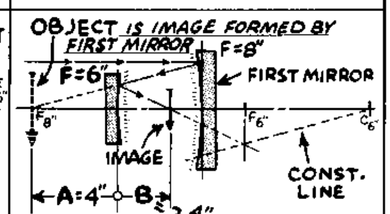
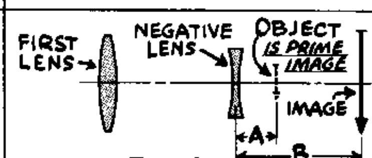
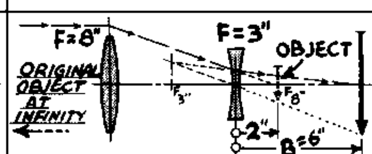
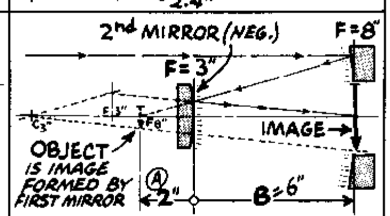
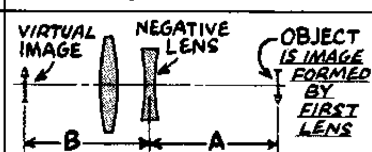
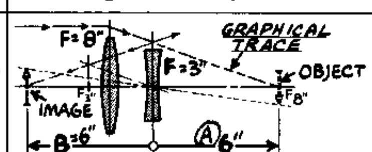
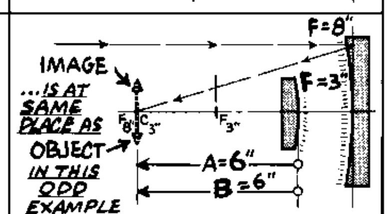
Graphical and math methods are compared in Figs. 1 and 2, which show a typical telephoto lens. A Barlow lens used with a telescope is similar. The graphical trace is done by the parallel ray method for the first lens; then, the same ray is put through the second lens with the oblique ray method. The diagram must be made

MATH vs Graphical Trace



③ OBJECT - IMAGE MATH

HOW TO FIND IMAGE DISTANCE from LENS WHEN OBJECT DISTANCE IS KNOWN

CASE NO.	OPTICAL SYSTEM	OBJECT DISTANCE	Formula for IMAGE DISTANCE	Example: LENS	Example: MIRROR
1	ANY POSITIVE LENS OR MIRROR	MORE THAN ONE F.L. BUT LESS THAN INFINITY	 $B = \frac{F \times A}{A - F}$	 $B = \frac{4 \times 6}{6 - 4} = \frac{24}{2} = 12''$	
2	ANY POSITIVE LENS OR MIRROR	LESS THAN ONE FOCAL LENGTH	 $B = \frac{F \times A}{F - A}$	 $B = \frac{4 \times 3}{4 - 3} = \frac{12}{1} = 12''$	
3	ANY NEGATIVE LENS OR MIRROR	ANY DISTANCE	 $B = \frac{F \times A}{F + A}$	 $B = \frac{6 \times 12}{6 + 12} = \frac{72}{18} = 4''$	
4	THE SECOND OF TWO POSITIVE LENSES OR MIRRORS	ANY DISTANCE	 $B = \frac{F \times A}{F + A}$	 $B = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = 2.4''$	
5	THE NEGATIVE LENS OR MIRROR OF A POS-NEG COMBINATION	LESS THAN ONE FOCAL LENGTH	 $B = \frac{F \times A}{F - A}$	 $B = \frac{3 \times 2}{3 - 2} = \frac{6}{1} = 6''$	
6	THE NEGATIVE LENS OR MIRROR OF A POS-NEG COMBINATION	MORE THAN ONE FOCAL LENGTH	 $B = \frac{F \times A}{A - F}$ (VIRTUAL IMAGE AT LEFT)	 $B = \frac{3 \times 6}{6 - 3} = \frac{18}{3} = 6''$ LEFT	

ALL CASES: **A** *is the OBJECT DISTANCE;* **B** *is the IMAGE DISTANCE. MAGNIFICATION (Linear):* $M = B/A$

fairly "fat," as shown, in order to obtain a reasonable degree of accuracy. The diagram can be made in less than 5 minutes when you are familiar with the procedure.

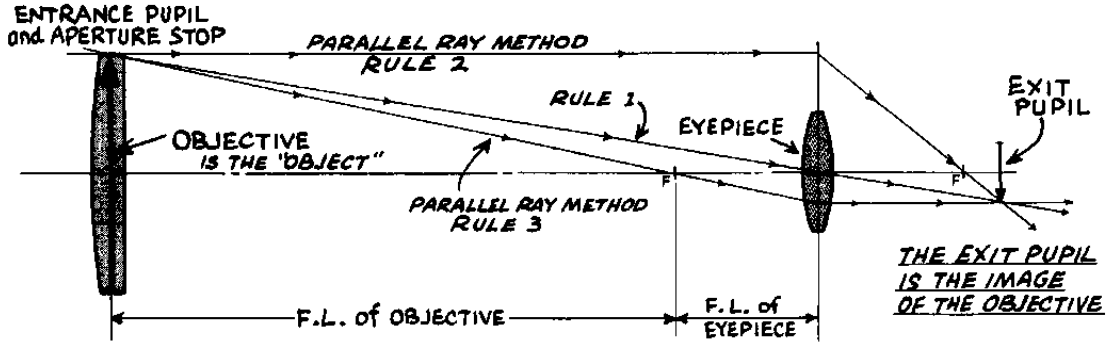
The math solution is done by finding the image position for the first lens and then using this as the object for the second lens. This locates the final image plane of a distant object, and this image position is also the focal plane of the combined lenses. The equivalent focal length is found by a separate calculation, as shown.

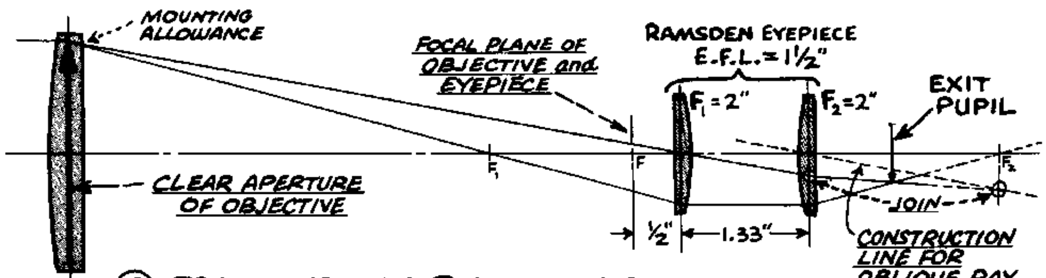
With an infinity object, you can also find e.f.l. by multiplying the linear M. of second lens by the f. l. of first lens. This is the formula commonly used for Barlow lenses. In the example shown, the linear M. of second lens is B/A is 6/2 is 3x. Then, 3 times 8 gives 24 inches for the focal length of the combined lenses. This calculation is used only for a distant object; if the object is at a lesser distance, the first lens will contribute to the magnification so that it is not an exclusive feature of the second lens.

STOPS and Pupils

① EXIT PUPIL of a TELESCOPE

GRAPHICAL TRACE TO FIND SIZE and POSITION





② TRACE THROUGH 2-LENS EYEPIECE

IN EVERY optical system there is one lens or diaphragm which limits the size of the bundle of light rays that can get through the lens system. This limiting aperture is called the aperture stop. Often it is the front lens of an instrument, which is the case for telescopes and microscopes. For a camera lens, the aperture stop is the iris diaphragm located between the lenses. When you use a magnifying glass, your eye is the smallest aperture in the optical system, and so becomes the aperture stop.

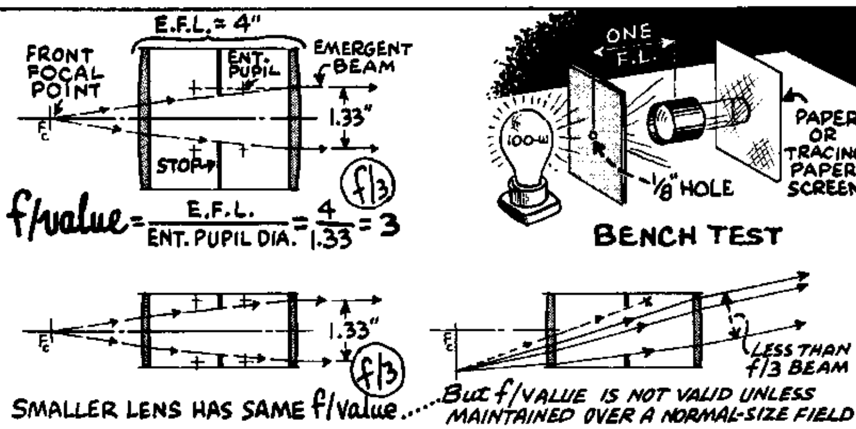
The pupils are images of the aperture stop. The entrance pupil is the image of the aperture stop formed by the lens or lenses ahead of it. If there is no lens in front of the aperture stop, the aperture stop itself assumes a dual role, being also the entrance pupil. Thus, in a telescope, the front lens is the aperture stop and and also the entrance pupil.

The exit pupil is the image of the aperture stop formed by the lens or lenses behind it. In the case of a telescope, the lens behind the objective is the eyepiece; the image of the objective formed by the eyepiece is the exit pupil. If the eyepiece is a single lens, the exit pupil is readily located by the parallel ray method, as shown in Fig. 1. More often the eyepiece is a two-lens system and the trace to locate the exit pupil is done by the parallel ray method for the first lens, and the oblique ray method for the second. Fig. 2 is an example. In astro telescopes with long objectives, the exit pupil is approximately one focal length of the eyepiece behind the eyepiece. In other words, the eyepiece is looking at a comparatively distant object (the objective), and so forms an image at about one f.l. In short telescopes and binoculars, the objective is closer to the eyepiece with the result the exit pupil will be found a little more than one focal length behind the eyepiece.

A DUPLET WITHOUT A STOP. One of the most familiar optical systems is the simple lens duplet, with or without a stop. When there is no

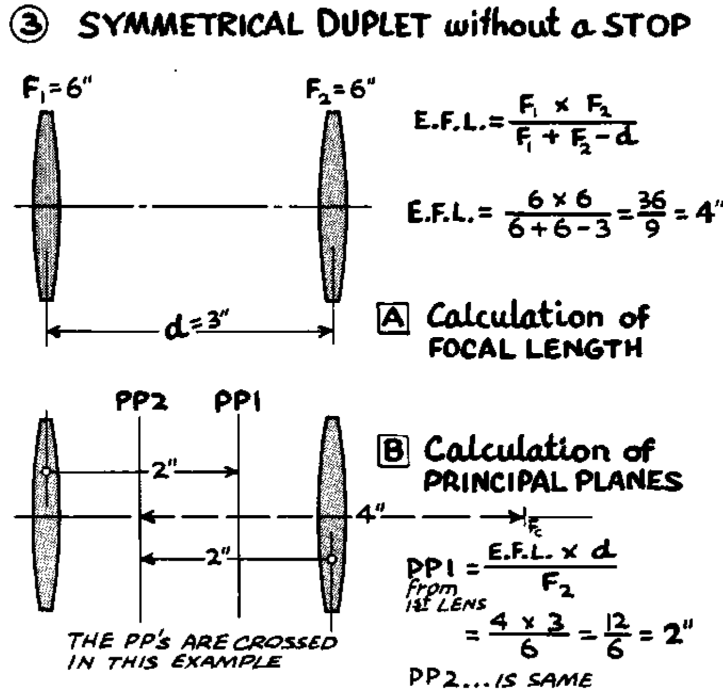
f/value of a DUPLET

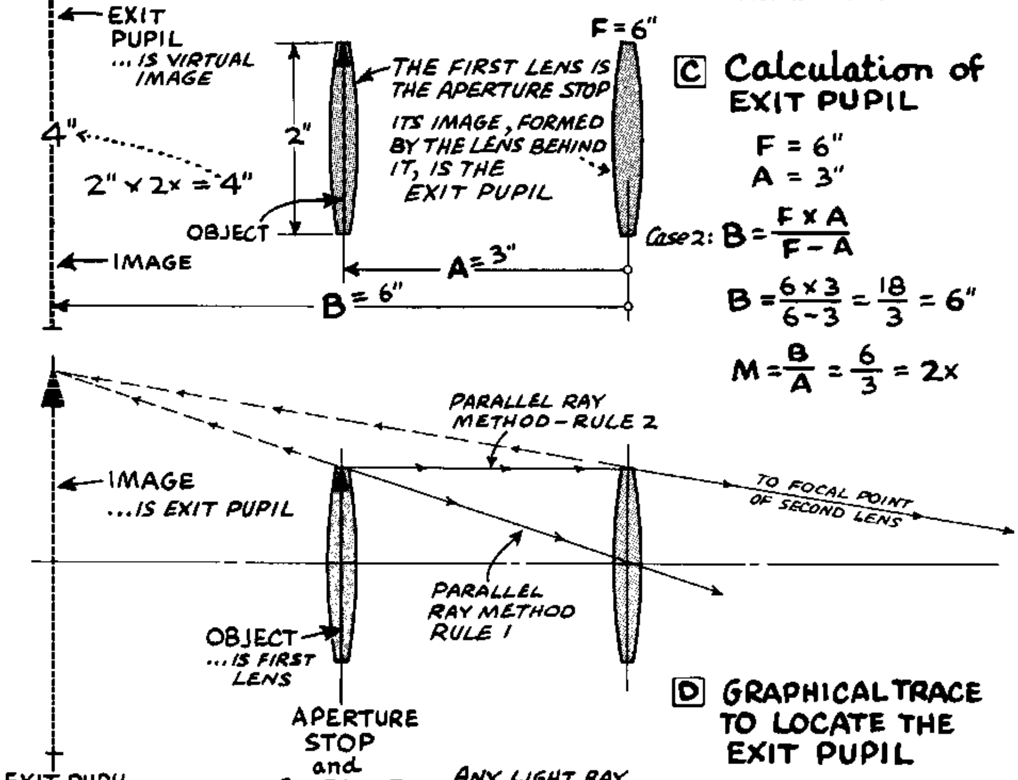
If you make a graphical trace through a duplet from the front focal point, the emergent parallel beam will be the diameter of the entrance pupil. Then, by the usual formula, the f/value of the lens is the focal length divided by the diameter of the entrance pupil. The f/value is not valid unless the lens covers some practical size of angular field, commonly 25 degrees or more.

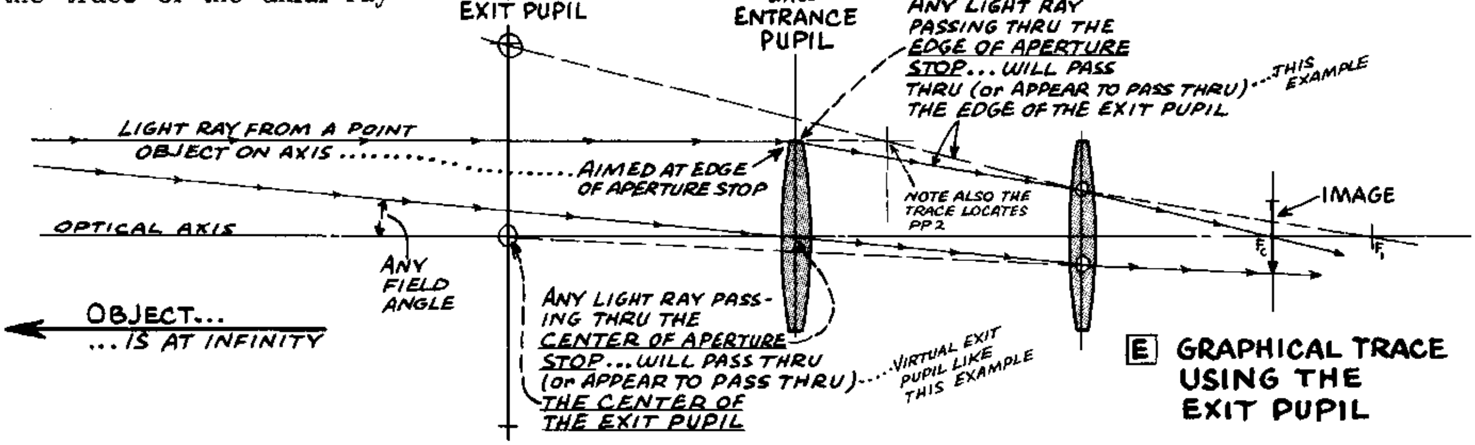


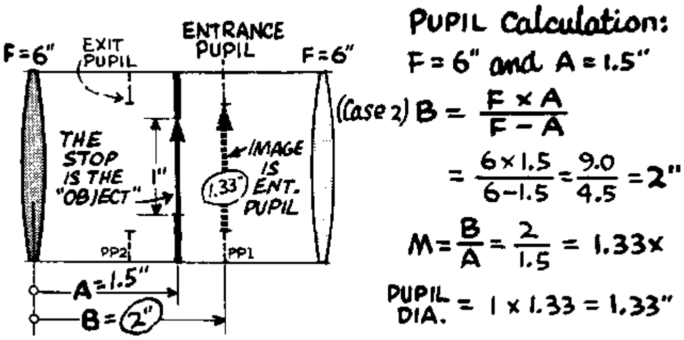
stop, the front lens is the aperture stop and also the entrance pupil. The exit pupil is the image of the first lens formed by the second lens. Preliminary date is usually obtained mathematically, using the simple formulas already described. The math work includes the e.f.l. and the PP's, Fig. 3, and may include the exit pupil itself, as shown at C. Unlike the telescope, the exit pupil is a virtual image, appearing to be ahead of the first lens, as shown. If desired, its position can be found graphically, as shown in Fig. 3D.

Once you know the location of the exit pupil, it offers yet another method of graphically tracing light rays through the optical system. Fig. 3E is a typical example, with the object located at infinity. The graphical trace of light rays through the first lens is done by the parallel ray method. You already know how to put the light ray through the second lens by using the oblique ray method. However, the exit pupil now offers an alternate method. Since the exit pupil is a picture of the first lens, anything that happens to the first lens will also happen to the image of the first lens, which is the exit pupil. So, if a light ray goes through the edge of the first lens, it will also pass through (or appear to pass through) the exit pupil. This particular example is a virtual image, so the refracted ray only appears to come from the edge of the exit pupil. However, the virtual exit pupil serves just as well as a real one in locating the ray path through the second lens. It can be seen in the drawing, the trace of the axial ray

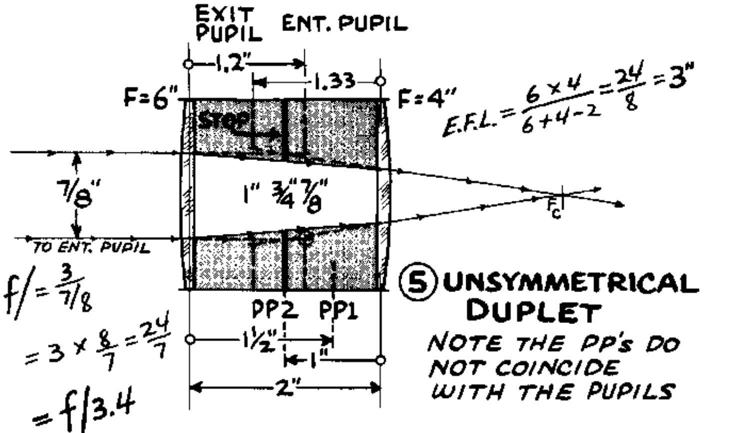


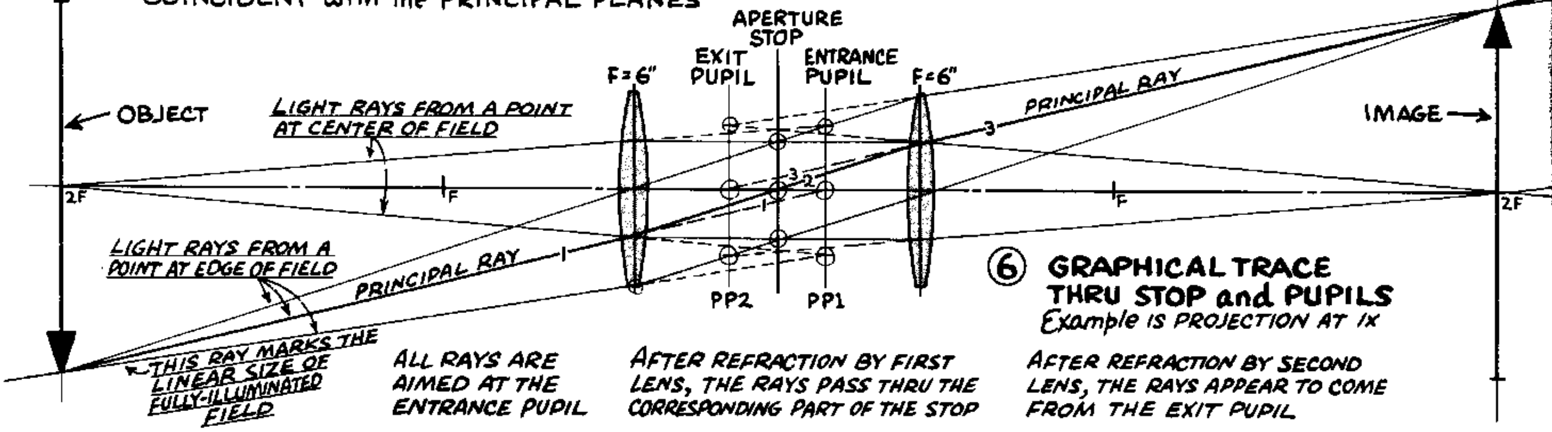






④ SYMMETRICAL DUPLET with a STOP
THE PUPILS ARE SYMMETRICAL and ARE COINCIDENT with the PRINCIPAL PLANES





⑥ GRAPHICAL TRACE THRU STOP and PUPILS
Example IS PROJECTION AT 1X

also locates the second principal plane as well as the rear focal point of the combined lenses.

With other graphical methods you can obtain good accuracy by drawing construction lines beyond the diameter of small lenses, but with the stop-and-pupil technique you must stick to the exact sizes of stop and pupils. This often means narrow angles, apt to be inaccurate in locating the image position. Hence, it is preferable to calculate the image position and then use the stop and pupils only to determine the exact path of the light rays through the lens.

DUPLET WITH A STOP. In this familiar system, the diaphragm is the aperture stop. The image of the stop formed by the first lens is the entrance pupil, while the image of the stop formed by the second lens is the exit pupil. The pupils are usually calculated. Since the average system is symmetrical, one calculation serves for both entrance and exit pupils, Fig. 4. However, if the duplet is not symmetrical, you will have different locations for the pupils, as shown in Fig. 5 example.

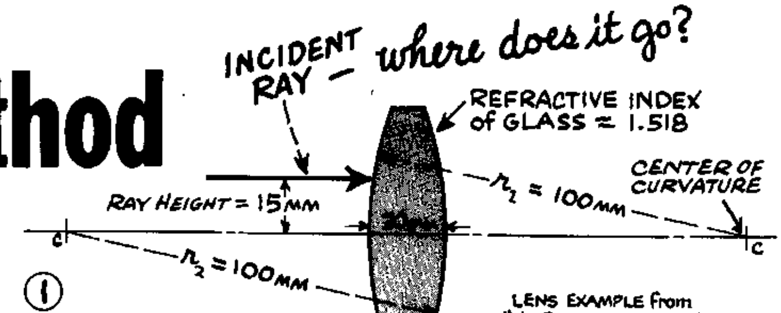
Fig. 6 shows how a ray trace is made using the stop and pupils. This is shown for 1x projection, which is a convenient and compact test case with both object and image at two focal lengths. The general method of making the ray trace is the same as already described. You know the entrance pupil is a picture of the stop formed by the first lens. So, you aim the light rays at the image of the stop, knowing that after refraction by the first lens the rays will then go through the stop itself. Similarly, when the light rays leave the system, they will seem to come from the image of the stop, which is the exit pupil. The rays through the center of the lens, Fig. 6, are easy to follow.

SIZE OF FIELD. The linear field of a duplet is determined by a line connecting the clear aperture of first lens with the lower edge of the entrance pupil; when extended to object plane, this line marks the limit of the fully-illuminated field.

Fig. 6 diagram reveals the general nature of the symmetrical duplet. Note that a point at edge of field gets just about as much light as a point right on the axis, making the overall lighting perfectly uniform. Note also how the edge-of-field rays criss-cross the two lenses, going through the thin part of one lens and the thick part of the other, a path which tends to eliminate the lateral faults of coma, distortion and color. You still get poor performance from the longitudinal faults, particularly spherical aberration, which can be decreased only by using a smaller stop. Depending on what it is used for, the simple lens duplet is rarely practical if faster than f/4.5.

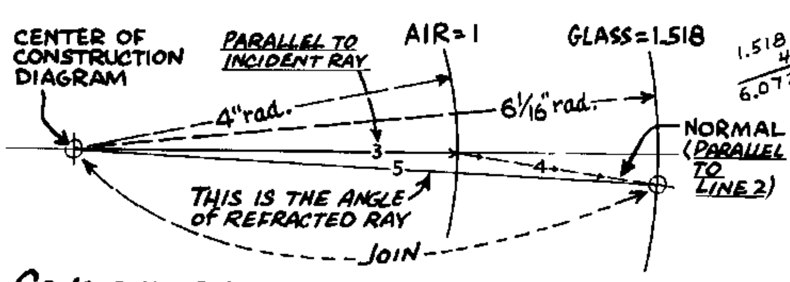
surface-by-surface

glass INDEX method

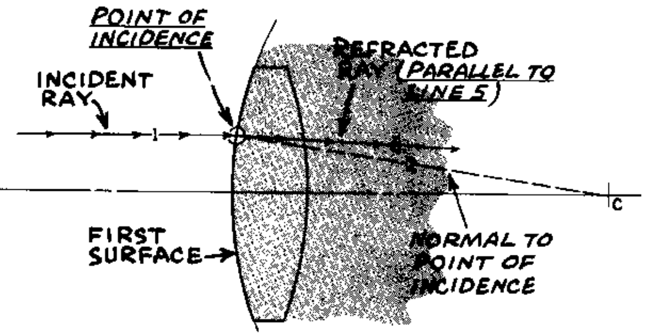


① TYPICAL PROBLEM and STARTING DATA

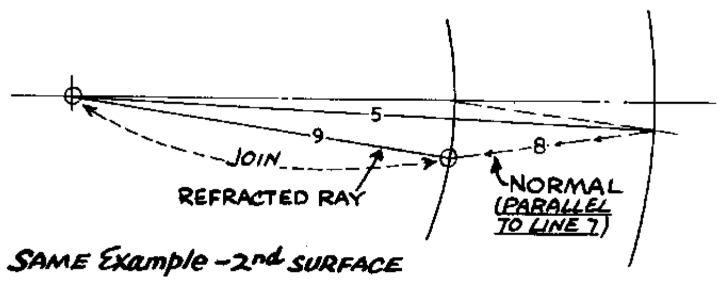
IT IS a needless refinement to use a surface-by-surface ray trace to solve a simple object-image problem, but if you want to find the spherical aberration of a lens system, the separate surface trace is the answer, whether you do the work by math or graph. The graphical method shown is a vector method applied to the refractive index of



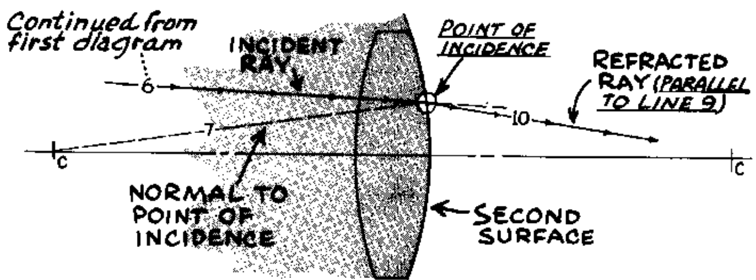
CONSTRUCTION DIAGRAM (½ FULL SIZE)
THE CIRCULAR ARCS REPRESENT THE INDEXES OF REFRACTION. USE ANY CONVENIENT SCALE, SUCH AS 4x (shown)



② TRACE THROUGH FIRST SURFACE



SAME Example – 2nd SURFACE



③ TRACE CONTINUED THRU SECOND SURFACE

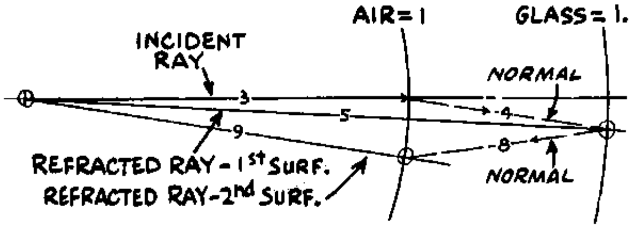
the glass. The trace is easy to do and makes an interesting study even if you have no great use for it.

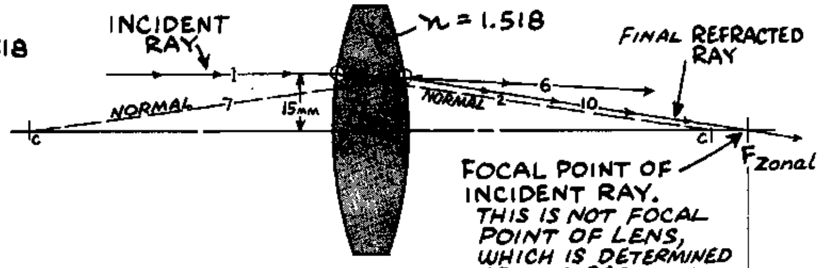
AUXILIARY CONSTRUCTION DIAGRAM. A separate construction diagram is needed in addition to a diagram of the lens itself. The construction diagram consists of two circular areas, one repsenting the refractive index of air, which is 1, while the other is the index of the glass used for the lens.

The sample problem, Fig. 1, is broken into two stages to show the process more clearly. Lines 1 and 2 are drawn first, these being on the lens diagram. The angle of line 2 is then transferred to the construction diagram, being line 4. Then, if you join the end of this line to the center of the construction diagram, you will get the angle of the refracted ray at the first surface. This angle is then transferred to the lens drawing.

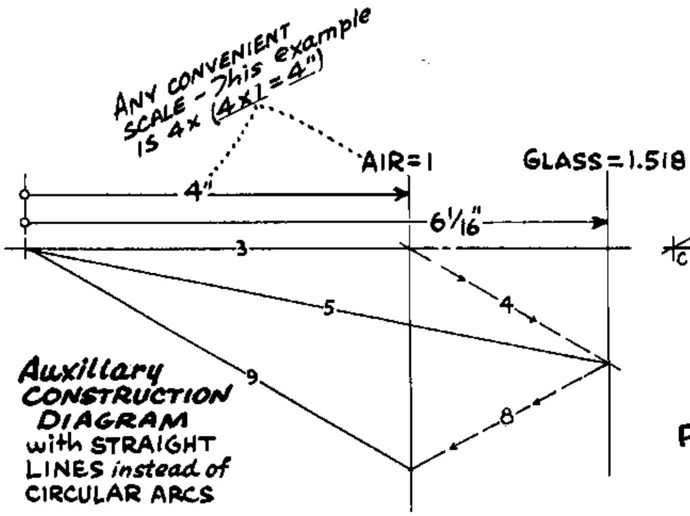
The process is repeated for the second or additional surfaces. The work is mainly a matter of transferring angles from one diagram to the other. This is readily done with an adjustable triangle or similar drafting tool.

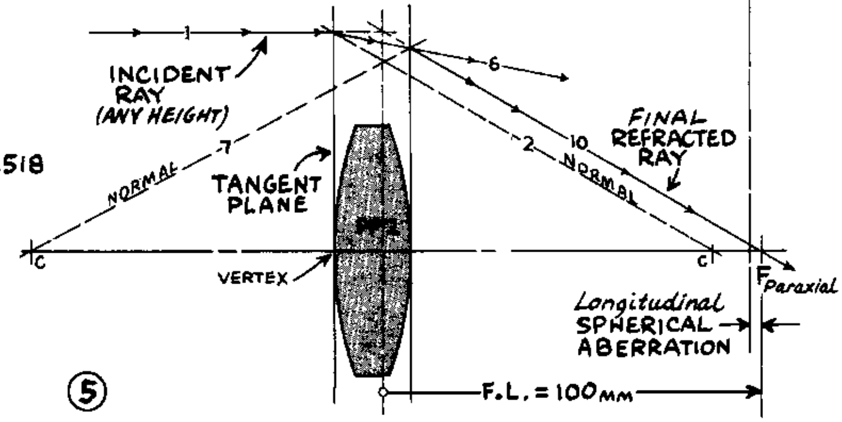
Fig. 4 shows the complete trace in one diagram. Where the light ray crosses the axis is the focal point for this particular light ray. The reference standard for the focal length of a lens is obtained from a paraxial (PAR-axial) ray, which means a light ray on or near the axis. If you were to draw such a ray, you would be likely to make a considerable error because of the slim angles involved. What you need is a paraxial trace, but drawn at some comfortable distance from the axis. This can be done by simply duplicating the conditions at the center of the lens, which is accomplished by using tangent planes, Fig. 5. Now you can work at any distance from the axis, and the light ray will perform





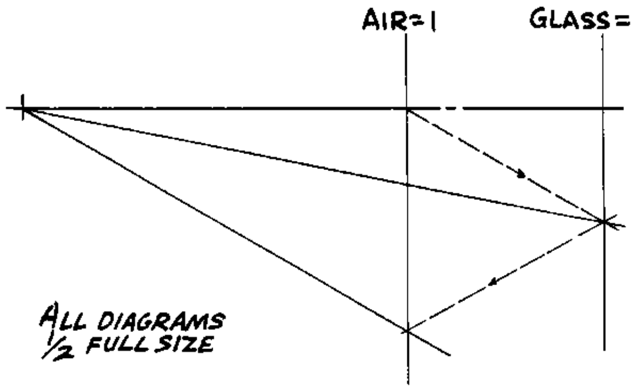
④ *SAME Example AS PREVIOUS PAGE, showing complete trace*

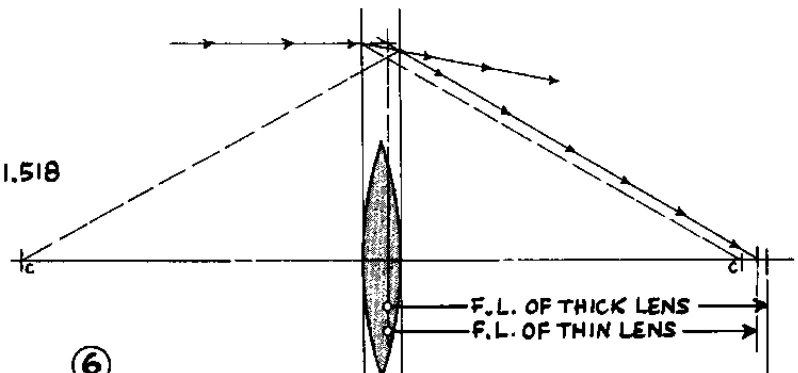




⑤

PARAXIAL TRACE LOCATES FOCAL PLANE and PP2

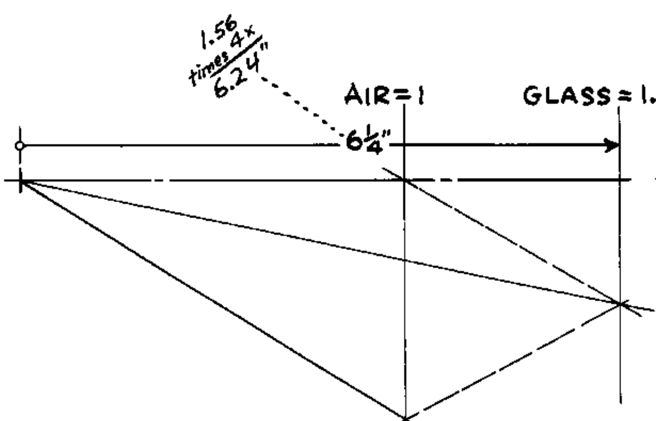


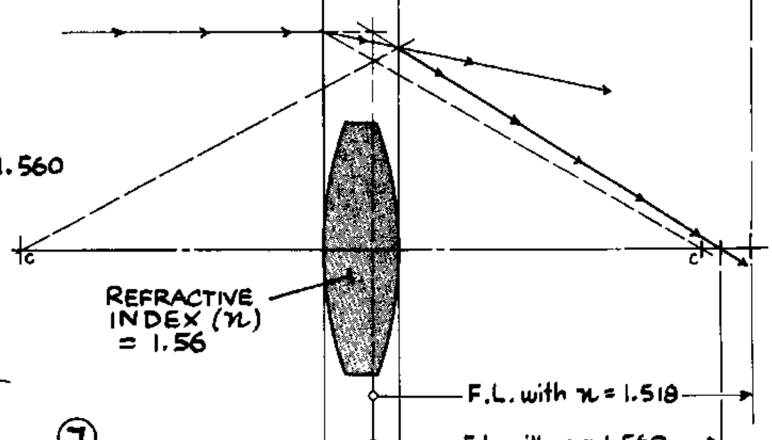


⑥

***Comparison:* SAME GLASS BUT THINNER**

FACT: *THICKER GLASS INCREASES THE FOCAL LENGTH*

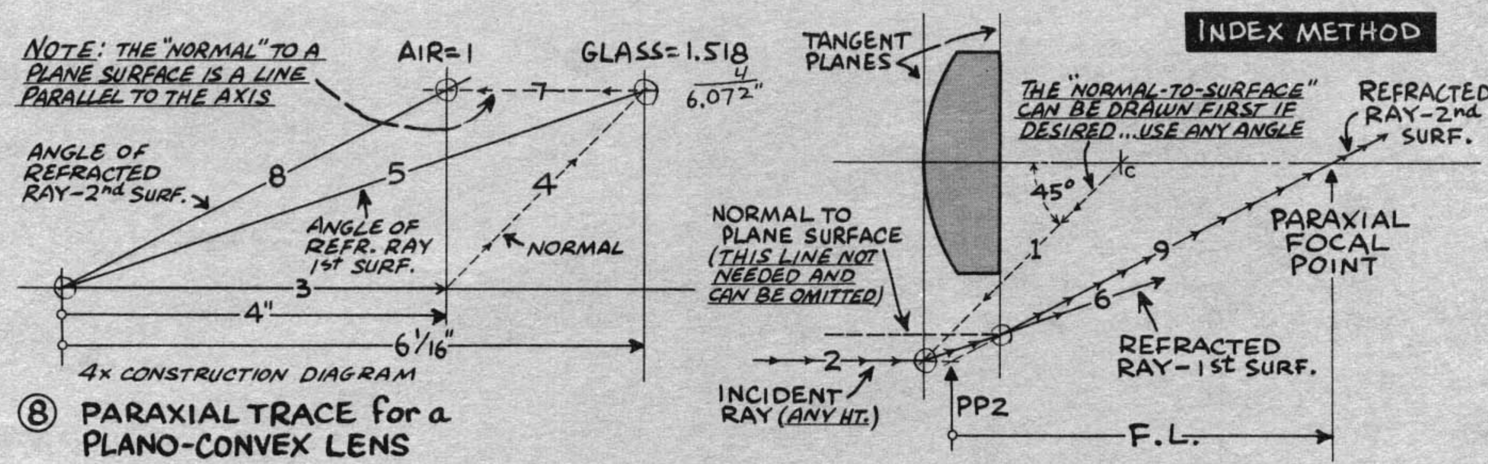




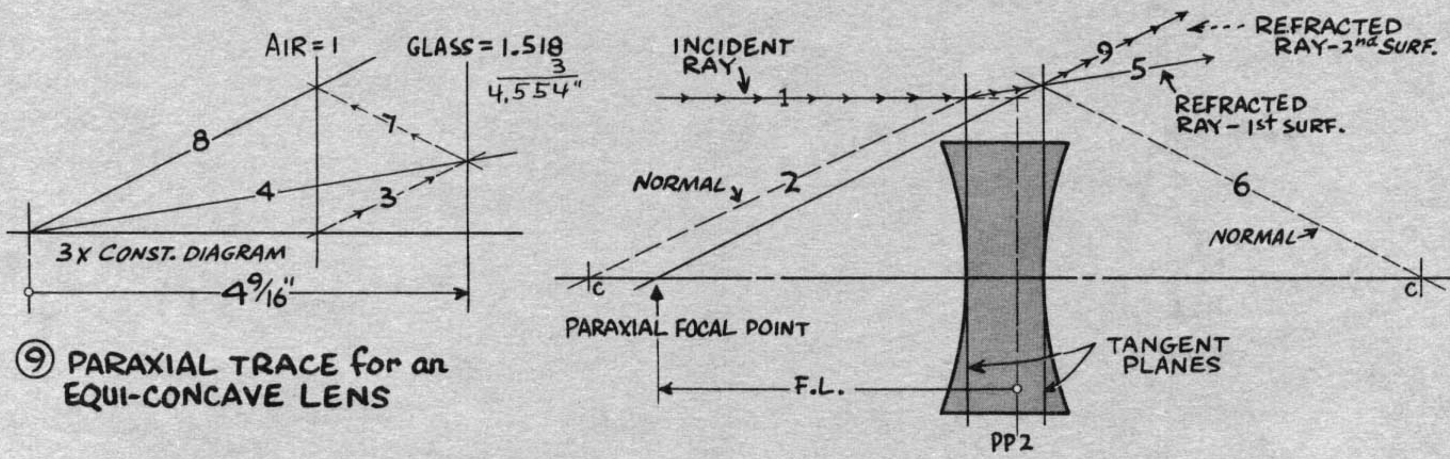
⑦

***Comparison:* SAME LENS BUT DIFFERENT GLASS**

FACT: *A HIGHER n NUMBER DECREASES THE F.L.*



⑧ **PARAXIAL TRACE for a PLANO-CONVEX LENS**



⑨ **PARAXIAL TRACE for an EQUI-CONCAVE LENS**

exactly like a paraxial ray. From the paraxial trace you get the focal plane of the lens, which is at the point where the light ray cuts across the axis.

SPHERICAL ABERRATION. A paraxial trace as described locates the focal point of the lens. Ideally, all rays should come to this same focus--any departure is the fault known as spherical aberration. Simple lenses will always show some S. A. It is always under-corrected for positive lenses, that is, a marginal ray will fall short of the paraxial focus. Spherical aberration increases approximately as the square of the ray height--a ray twice as far from the axis will have four times the S. A.

To find the S. A. of a lens, you make a paraxial trace using tangent planes to find the paraxial focus. Then, a marginal ray is drawn. It can be the same incident parallel ray as before, but this time it is drawn to the surface of the lens, and the construction diagram uses index circles instead of straight lines. The difference in the axial intercept is the longitudinal spherical aberration. Unfortunately, you can't use this method for mirrors for the simple reason the "index" of a mirror is 1, the same as air.

VARIATIONS IN FOCAL LENGTH. The various graph and math methods used to find f. l. may vary slightly since some of the methods are merely close approximations. For example, the f.l. of an equi-convex lens is the same as its radius of curvature--F=R. Neither glass thickness nor index are considered.

Fig. 6 is graphical proof that the f.l. of a lens is increased by glass thickness. Fig. 7 shows that a high index number will decrease the focal length. Simple lens formulas are based on an assumed index of 1.50. However, the most common optical glass has an index of about 1.52. And, of course, even a thin lens has some glass thickness. The next result is that the two variables are more or less self-compensating.

OTHER EXAMPLES. Fig. 8 shows a paraxial trace through a plano-convex lens. It can be seen that the "normal" to a plane surface is a line parallel with the axis. Another small variation here is the normal to first surface, which, if desired, can be drawn first at any convenient angle, here 45 degrees.

Fig. 9 shows the paraxial trace through a negative lens. It can be seen the trace also serves to locate the principal planes, which are symmetrical in a symmetrical lens.